Vacuum Permittivity and General Relativity

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ABSTRACT

Vacuum permittivity is the scalar in Maxwell's equations that determines the speed of light and the strength of electrical fields. In 1907, Einstein showed that vacuum permittivity changes when inertial reference frames accelerate. In 1952, Møller generalized Einstein's result by proving that vacuum permittivity changes with the curvature of static spacetimes in general relativity. In 1994, Sumner proved that vacuum permittivity changes with the curvature of Friedmann spacetime. Change in vacuum permittivity shifts the energies of both photons and atomic emissions. While photon energies are proportional to vacuum permittivity, atomic emissions are proportional to the square of vacuum permittivity. This alters the interpretation of Hubble redshift. Hubble redshift is observed when a photon emitted by an atom long ago is redder than the same one emitted today on earth. This occurs only when a closed Friedmann universe is accelerating in collapse, when atomic emissions have out blueshifted photons giving the relative redshift observed. This theoretical conclusion is confirmed by fitting the Pantheon redshift data of 1048 supernovas using only two free variables, a negative Hubble constant $H_o =$ -72.10 $\pm$ 0.75 km s^{-1} Mpc^{-1} and a nearly flat deceleration parameter $1/2 < q_o < 0.51$. There is no need for a cosmological constant. The acceleration in Hubble redshift measurements simply reflects the accelerating collapse of our Friedmann universe. The speed of light in Friedmann geometry is inversely proportional to the radius of the universe. It was infinite at the Big Bang, passing through its current value c at 9.05×10^9 years on its way to a minimum at full expansion. Now the speed of light is accelerating towards infinity as the universe collapses. In Schwarzschild geometry, the speed of light today is close to c at large distances from the mass but is zero at the Schwarzschild radius near the mass. Traditional analyses that explain Schwarzschild redshifts using time dilation agree with the curvature-dependent vacuum permittivity reasoning presented here.

1. Introduction

Einstein (1907) summarized the status of special relativity and its implications in the two years since its publication. At the end of his survey he concluded with a speculative section on

"The Principle of Relativity and Gravitation." He considered an uniformly accelerated coordinate system and assumed that locally it is equivalent to a gravitational field. Einstein determined that Maxwell's equations in the accelerated coordinate system (and hence in a gravitational field) are exactly the same as they are in the inertial coordinate systems of special relativity, except that "The principle of the constancy of the velocity of light does not hold ... the velocity of light in the gravitational field is a function of place ..." (Einstein 1911) [1]. This is also true in general relativity.

In Maxwell's equations, vacuum permittivity ε is the scalar that determines the speed of light and the strength of electrical fields. Einstein's discovery means that both the wavelengths of photons and the wavelengths of photons emitted by atoms change with gravity in special relativity.

Vacuum permittivity also changes with spacetime curvature in general relativity. Two exact solutions to the theory of general relativity are examined in this article, Schwarzschild's static solution for a spherical mass in an otherwise flat universe and Friedmann's closed, matter-filled, homogeneous universe.

In Schwarzschild spacetime, ε is a function of the distance r from the central mass. At large distances, $\varepsilon(r)$ is close to its flat space value but increases as r gets smaller. Atomic sizes and the wavelengths of the photons they emit both change with ε, but wavelengths of photons change by a smaller amount. The redshift equation derived here using general relativity and the traditional one assuming special relativity and time dilation give the same results despite making different assumptions.

In Friedmann spacetime, ε is directly proportional to the Friedmann radius. As the size of the Friedmann universe evolves, the changing strength of the electrical force between charges shifts atomic energy levels, changing the wavelengths of light atoms emit. Like atomic emissions, photons also change but by a smaller amount. This difference in evolution of atomic emissions and photons reverses the interpretation of Hubble redshift. The Friedmann universe is closed and collapsing, Figure 1.

Together the changes in atoms and photons replicate the acceleration measured in recent Hubble redshift observations. Identical changes in atoms and photons discribed here were found by Schrödinger (1939) who proved that quantum wave functions expand and contract exactly like the radius of a closed Friedmann universe.

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[1] English translation (Einstein 1993, p 385)

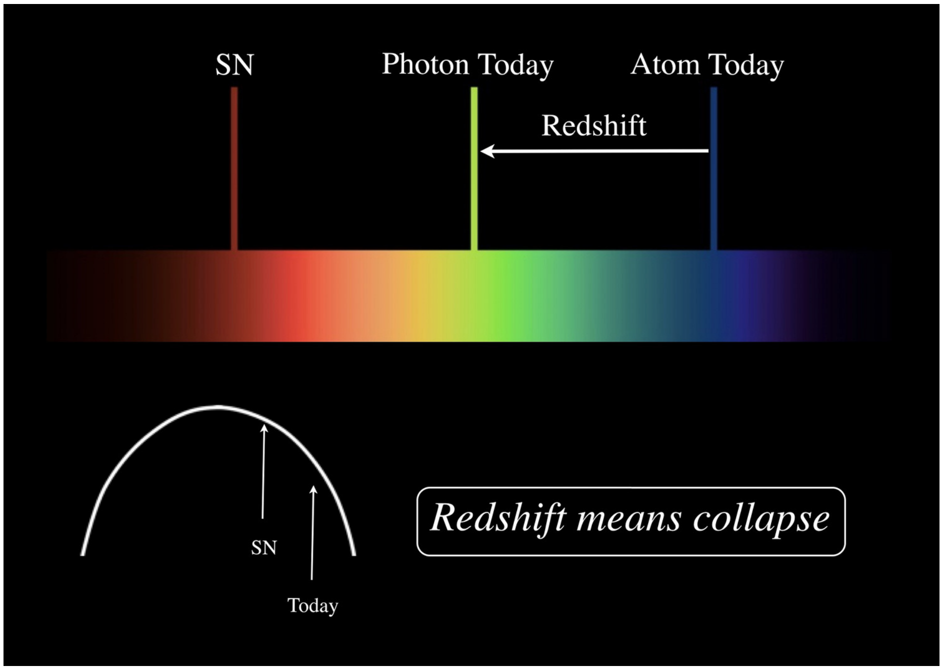

Fig. 1.— Photons blueshift in a collapsing universe. Atomic spectra blueshift more. Hubble redshift of the photons from the supernova (SN) is observed.

2. Mathematical Models

2.1. Einstein's Solution

In his study of Maxwell's equations in an uniformly accelerated coordinate system, Einstein (1907)[2] concluded that the velocity of light in special relativity, c, is reduced to c^*, the local coordinate velocity of light in the accelerated system. Einstein (1989, p 310) found in an accelerated system which corresponds locally to the gravitational field of a point mass

$$c^* = c\left(1 + \frac{\Phi}{c^2}\right), \tag{1}$$

where Φ is the Newtonian gravitational potential

$$\Phi = -\frac{k\,m}{r}. \tag{2}$$

m is the mass of the object creating the gravitational field at a distance r. k is the gravitational constant.

[2]English translation (Einstein 1989, p 252)

The connection between Einstein's result, equation (1), and the strength of the electrical field comes from the definition of relative vacuum permittivity ε,

$$\varepsilon \;=\; \frac{c}{c^{*}}. \tag{3}$$

Combining equations (1), (2), and (3) gives Einstein's value for ε[3]

$$\varepsilon\,(r) \;=\; \frac{1}{\left(1 \;-\; \dfrac{km}{rc^2}\right)}. \tag{4}$$

2.2. Schwarzschild Solution

The Schwarzschild solution in general relativity may be written (Landau & Lifshitz 1975, p 301)

$$\begin{aligned} ds^2 \;=\; \left(1 - \frac{2km}{rc^2}\right) c^2dt^2 \;-\; \frac{dr^2}{\left(1 - \dfrac{2km}{rc^2}\right)} \\ -\; r^2(d\theta^2 \;+\; sin^2\theta\, d\phi^2). \end{aligned} \tag{5}$$

Møller (1952, p 308) and Landau & Lifshitz (1975, p 258) studied the effects of curved spacetime on Maxwell's equations. Both proved that in a static gravitational field the electromagnetic field equations take the form of Maxwell's phenomenological equations in a medium at rest with

$$\varepsilon(r) = \frac{1}{\sqrt{g_{00}}}. \tag{6}$$

g_{00} is the time component of the metric tensor $g_{\mu\nu}$. [4]

Einstein's pre-general relativity result, equation (4), is the first approximation to the exact relativistic equations (5) and (6),

$$\varepsilon(r) = \frac{1}{\sqrt{g_{00}}} = \frac{1}{\sqrt{\left(1 - \dfrac{2km}{rc^2}\right)}} \;\approx\; \frac{1}{\left(1 \;-\; \dfrac{km}{rc^2}\right)}. \tag{7}$$

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[3]Vacuum permittivity is defined by NIST to be constant with the value $\varepsilon_o \;=\; 8.854187817\ldots x10^{-12}$ Farads/meter. But as Einstein showed, vacuum permittivity is not constant. When the numerical value for vacuum permittivity for a curved spacetime is needed, the product $\varepsilon \times \varepsilon_o$ is used with ε_o expressed in appropriate units. ε is often called relative vacuum permittivity.

[4]This result is valid for every static metric, not only the Schwarzschild metric.

2.3. Friedmann Solution

Friedmann (1922) published a closed universe solution to Einstein's theory of general relativity without a cosmological constant. The Friedmann universe rapidly expands from a singularity, slowing until it reaches a maximum size before accelerating back to a singularity.

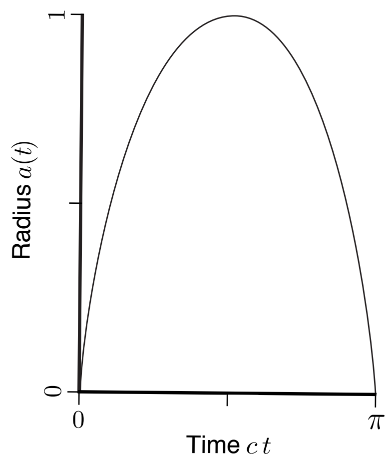

Fig. 2.— Friedmann's solution for a closed universe with $\alpha = 1$ in equation 9.

Friedmann assumed the metric,

$$ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dr^2}{(1-r^2)} + r^2\left(d\theta^2 + sin^2\theta\, d\phi^2\right)\right], \tag{8}$$

and homogeneous, incoherent matter, conserved in amount and exerting negligible pressure. His solution is the cycloid shown in Figure 2,

$$a = \frac{\alpha}{2}(1-\cos\psi) \qquad c\,t = \frac{\alpha}{2}(\psi - \sin\psi), \tag{9}$$

where α is a constant and $0 \le \psi \le 2\pi$ (Tolman 1934).

Sumner (1994) examined Maxwell's equations in Friedmann geometry and found that ε changes in time along with spacetime curvature,

$$\varepsilon\,(t) = a(t). \tag{10}$$

$a(t)$ is the radius of the Friedmann universe defined above with $\alpha = 1$.

2.4. Local Mathematical Coordinates

The effects of spacetime curvature in nature are often explained as gravitational forces or simply ignored. The extraordinary success of the special theory of relativity confirms this approach. But spacetime is never precisely flat as ubiquitous gravity clearly shows. Flat spacetime exists only in mathematical models. Every spacetime in nature is curved.

To understand the effects of spacetime curvature on atoms and photons, a coordinate system that includes spacetime curvature is necessary. The method used by Einstein, Møller, Landau, Lifshitz, and Sumner is adopted where a local pseudo-Cartesian coordinate system is used with the vacuum permittivity $\varepsilon(x^\mu)$ determined by the general relativistic geometry at that spacetime point. Specifically,

$$ds^2 = \frac{c^2}{\varepsilon^2(x^\mu)} dt^2 - \left[dr^2 + r^2 \left(d\theta^2 + sin^2\theta \, d\phi^2\right)\right]. \tag{11}$$

If the variation in $\varepsilon(x^\mu)$ in the region of interest is ignored, equation (11) is just the metric of special relativity with a velocity of light $c/\varepsilon(x^\mu)$. If $\varepsilon(x^\mu) = 1$ the result is special relativity with spacetime curvature ignored.

2.5. Changes in Atoms and Photons

Photon wavelengths, atomic sizes, and the wavelengths of photons atoms emit change with ε. In the following equations κ (the Greek letter kappa) is to be replaced by either the radial coordinate r for Schwarzschild geometry or the time coordinate t for Friedmann geometry. The logic and math are the same for each geometry.

The Bohr radius a_o of a hydrogen atom in its ground state at κ is [5]

$$a_o(\kappa) = \frac{4\pi\varepsilon_o \, \varepsilon\,(\kappa)\, \hbar^2}{me^2}. \tag{12}$$

$\varepsilon_o = 8.854187817\ldots x10^{-12}$ F/m (farads per meter) is the defined value of ε_o. m is the mass of the electron, e is the charge of the electron, and $\hbar$ is Planck's constant h divided by 2π. These are assumed to remain constant as spacetime curvature changes.

The change in Bohr radius a_o as κ changes is

$$\frac{a_o(\kappa_1)}{a_o(\kappa_2)} = \frac{\varepsilon\,(\kappa_1)}{\varepsilon\,(\kappa_2)}. \tag{13}$$

The characteristic wavelength λ_e emitted by a hydrogen atom during the transition between the principle quantum numbers n_2 and n_1 is

$$\lambda_e(\kappa) = \frac{8\,\varepsilon_o^2\,\varepsilon^2\,(\kappa)\,h^3\,c}{me^4} \left(\frac{n_1^2\,n_2^2}{n_2^2 - n_1^2}\right). \tag{14}$$

[5]See standard texts, e.g. Leighton (1959).

c in equation (14) comes from the defining relationship between λ and ν, $\lambda\,\nu\,=\,c$.

The change in $\lambda_e(\kappa)$ as κ changes is

$$\frac{\lambda_e(\kappa_1)}{\lambda_e(\kappa_2)} = \frac{\varepsilon^2\,(\kappa_1)}{\varepsilon^2\,(\kappa_2)}. \tag{15}$$

Consider the Compton wavelength, λ_c, of a particle with mass m_p,

$$\lambda_c(\kappa) = \frac{h}{m_p\,c^*(\kappa)} = \frac{h\,\varepsilon\,(\kappa)}{m_p\,c}. \tag{16}$$

The change in $\lambda_c(\kappa)$ as κ changes is

$$\frac{\lambda_c(\kappa_1)}{\lambda_c(\kappa_2)} = \frac{\varepsilon\,(\kappa_1)}{\varepsilon\,(\kappa_2)}. \tag{17}$$

The Compton wavelength of a particle is equivalent to the wavelength of a photon of the same energy as the particle. Compton and photon wavelengths have the same $\varepsilon(\kappa)$ dependency that the Bohr radius has. The wavelength change for a photon is

$$\frac{\lambda(\kappa_1)}{\lambda(\kappa_2)} = \frac{\varepsilon\,(\kappa_1)}{\varepsilon\,(\kappa_2)}. \tag{18}$$

2.6. Gravitational Redshift

The following notation is used. The wavelength of a photon λ emitted at κ_1 and examined at κ_1 will be written $\lambda(\kappa_1, \kappa_1)$. The wavelength of a photon λ emitted at κ_1 and examined at κ_2 will be written $\lambda(\kappa_1, \kappa_2)$.

The traditional redshift z formula assumes that atomic emissions do not evolve, $\lambda(\kappa_2, \kappa_2) = \lambda(\kappa_1, \kappa_1)$, but that photons do (equation (18)).

$$z\;=\;\frac{\lambda(\kappa_1, \kappa_2)\;-\;\lambda(\kappa_1, \kappa_1)}{\lambda(\kappa_1, \kappa_1)}\;=\;\frac{\varepsilon(\kappa_2)}{\varepsilon(\kappa_1)}\;-\;1. \tag{19}$$

κ_2 is the observer's location and κ_1 is the location at the time of emission.

Since atomic emissions do evolve with spacetime geometry, a new redshift variable ζ (the Greek letter zeta) is defined to match what is done experimentally,

$$\zeta\;=\;\frac{\lambda(\kappa_1, \kappa_2)\;-\;\lambda(\kappa_2, \kappa_2)}{\lambda(\kappa_2, \kappa_2)}\;=\;\frac{\varepsilon(\kappa_1)}{\varepsilon(\kappa_2)}\;-\;1. \tag{20}$$

κ_2 is the observer's location and κ_1 is the location at the time of emission.

2.7. Schwarzschild Redshift

The Schwarzschild metric describes the spacetime geometry around a spherical mass. Substituting $\varepsilon(r)$ from equation (7) into equation (20) gives

$$\zeta = \frac{\sqrt{\left(1 - \dfrac{2km}{r_2 c^2}\right)}}{\sqrt{\left(1 - \dfrac{2km}{r_1 c^2}\right)}} - 1. \tag{21}$$

This equation is the same as traditional derivations using time dilation with the Schwarzschild metric. See e.g. Weinberg (1972, p 80). Every experiment that confirms equation (21) confirms the reasoning presented here.

For a source and a receiver on earth separated by a small distance h, the weak field approximation to this exact equation is

$$\frac{\Delta\nu}{\nu} \approx \frac{km}{r^2 c^2} h, \tag{22}$$

where ν is the frequency of the photon emitted, k is the gravitational constant, m is the mass of the earth, r is the distance from the center of the earth to the source, and c is the speed of light. [6]

2.8. Friedmann Redshift

Substituting $\varepsilon(t) = a(t)$ into equation (20) gives the redshift ζ for Friedmann geometry,

$$\zeta = \frac{a(t_1)}{a(t_2)} - 1. \tag{23}$$

Hubble redshift ($\zeta > 0$) implies $a(t_1) > a(t_2)$. The universe was larger in the past, $a(t_1)$, than it is now, $a(t_2)$. This puts us somewhere on the collapsing half of the curve in Figure 2. The logic is simple. Since Hubble shifts are red ($\zeta > 0$), the Friedmann universe is collapsing. If Hubble shifts were blue ($\zeta < 0$), the Friedmann universe would be expanding.

3. Analyzing Hubble Redshifts

The analysis of redshift observations must include the changes in atomic emissions in addition to the changes in photons. Astronomers measure the redshift defined by ζ, equation (23). The

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[6] Will (2014, p 15) has a good explanation how this approximate equation can be derived without general relativity.

following derivation is similar to the one made when atomic evolution is ignored and the universe is assumed to be expanding (Narlikar 1983), but is different because ζ not z describes the observed redshift and some choices in signs are made differently when the universe is contracting (Sumner & Vityaev 2000). It is assumed that observed photons were emitted after contraction began.

The mathematical coordinate distance r to a source can be shown to be a function of the observed redshift ζ of the source and the deceleration parameter q_o in the following way.

Setting $ds = 0$ in the Friedmann metric, equation (8), gives

$$c\,dt = \frac{-a(t)\,dr}{(1 - r^2)^{1/2}}. \tag{24}$$

The source is located at the spatial coordinates $(r_1, 0, 0)$ with emission at time t_1 and the observer is at $(0, 0, 0)$ with reception at time t_2.

$$c \int_{t_1}^{t_2} \frac{dt}{a(t)} = \int_0^{r_1} \frac{dr}{(1 - r^2)^{1/2}} = \sin^{-1} r_1. \tag{25}$$

Substituting $a(t)$ and dt calculated from the Friedmann solution, equation (9), gives

$$r_1 = \sin(\psi_2 - \psi_1). \tag{26}$$

The Friedmann equation for the closed universe is (Narlikar 1983, p 113)

$$\dot{a}^2 = c^2 \left(\frac{\alpha}{a} - 1 \right). \tag{27}$$

The Hubble constant H and the deceleration parameter q are defined by

$$H(t) = \frac{\dot{a}(t)}{a(t)}, \qquad \frac{\ddot{a}(t)}{a(t)} = -q(t) H^2(t). \tag{28}$$

" $\cdot$ " indicates time derivative. H is negative and q is greater than 1/2 for a closed, collapsing universe. Present day values are denoted by H_o and q_o.

α, the constant in equations (9), may be written (Narlikar 1983, p 114)

$$\alpha = \frac{2q_o}{(2q_o - 1)^{3/2}} \frac{c}{|H_o|}. \tag{29}$$

Solving for ψ_2 and ψ_1 in terms of ζ and q_o and substituting into equation (26) gives

$$r_1 = \frac{(2q_o - 1)^{1/2}}{q_o} \left[\zeta - \frac{(1 + \zeta)(1 - q_o)}{q_o} \right] + \frac{(1 - q_o)}{q_o} \left\{ 1 - \left[\zeta - \frac{(1 + \zeta)(1 - q_o)}{q_o} \right]^2 \right\}^{1/2}. \tag{30}$$

The flux f of photons is related to the luminosity L of the source and to its luminosity distance D_L by the equation

$$f \;=\; \frac{L}{4\pi D_L^2}. \tag{31}$$

D_L is determined in the following way. Calculate the observed flux f by noting that L, the actual luminosity of the source, is changed by a factor of $a(t_2)/a(t_1)$ because of the apparent change of the photon's energy and changed by another factor of $a(t_2)/a(t_1)$ because of the changes in time in the local metric, equation (11). The distance to the source is $r_1\,a(t_2)$. This gives an observed flux of

$$f \;=\; \frac{L\dfrac{a^2(t_2)}{a^2(t_1)}}{4\pi r_1^2 a^2(t_2)}. \tag{32}$$

Combining equations (31) and (32) using (23) gives

$$D_L \;=\; r_1\,a(t_2)\,(1\;+\;\zeta). \tag{33}$$

$a(t_2)$ is (Narlikar 1983, p 114)

$$a(t_2) \;=\; \frac{-c}{H_o}\,\frac{1}{(2q_o\;-\;1)^{1/2}}. \tag{34}$$

Substituting equations (30) and (34) into (33) gives

$$D_L \;=\; \frac{-c}{H_o}\frac{(1+\zeta)}{q_o}\left\{\left[\zeta - \frac{(1+\zeta)(1-q_o)}{q_o}\right] + \right.$$
$$\left.\frac{(1-q_o)}{(2q_o-1)^{1/2}}\left(1-\left[\zeta-\frac{(1+\zeta)(1-q_o)}{q_o}\right]^2\right)^{1/2}\right\}. \tag{35}$$

The relationship between distance modulus (the difference between the apparent magnitude m and absolute magnitude M of a celestial object) and luminosity distance, D_L, is

$$m - M = 5\log_{10}\left(\frac{D_L}{10\,\text{parsecs}}\right). \tag{36}$$

The Hubble constant H_o (negative for the contracting half of the curve) and the deceleration parameter q_o (which must be $> 1/2$ characterizing a closed Friedmann universe) are then varied to find best least-squared fits to Hubble redshift observations of ζ and $m - M$ using equations (35) and (36).

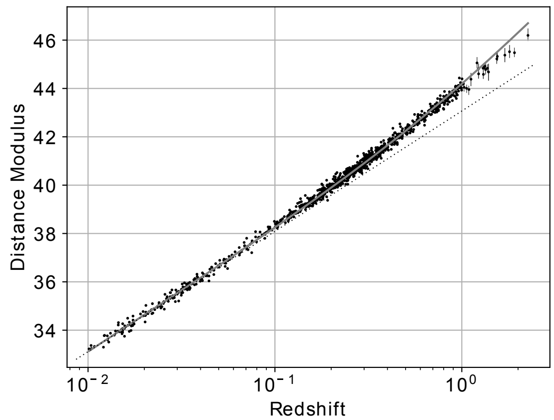

Fig. 3.— The solid line is the fit to the Pantheon redshift data with the parameters H_o = -72.10 $\pm$ 0.75 km s^{-1} Mpc^{-1} and $1/2 < q_o < 0.51$. The dotted straight line is included to visually clarify the upward curve (or "acceleration") of the data and fit. The average data error is 0.1418. The standard deviation for this fit is 0.1515.

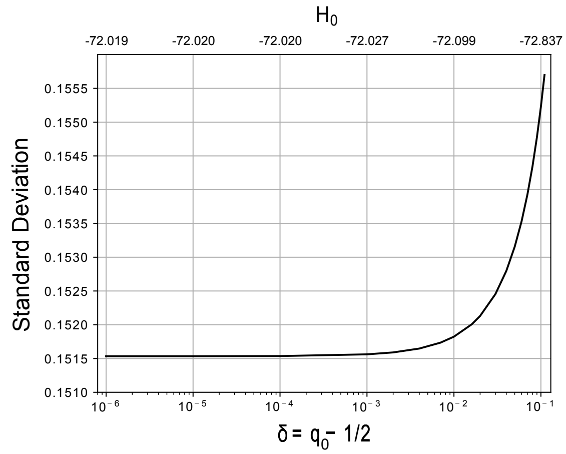

Fig. 4.— Standard deviation for fits at smaller values of $\delta = (q_o - 1/2)$. The values of H_o on the top axis are the best fits for the δ values on the bottom axis. No minimum for δ was found.

4. Fit to Pantheon SN Redshift Data

The Pantheon redshift data of 1048 supernovas (Scolnic 2018) were analyzed assuming that both atoms and photons change. The Hubble constant and deceleration parameter were the only variables, see Figures 3 and 4.

Since this Friedmann universe is closed, $q_o > 1/2$. Every search conducted found a lower standard deviation when q_o was closer to 1/2. No lower limit for $\delta = (q_o - 1/2)$ was found and the upper limit 0.51 was chosen because there is little change in the quality of fit with smaller q_o, hence $1/2 < q_o < 0.51$. This is illustrated in Figure 4.

5. Our Universe

$\frac{2}{3}\,|H_o|^{-1}$ estimates the time until collapse, t_c, of the Friedmann universe when q_o is this close to 1/2. For $H_o = -72.10\,km\,s^{-1}Mpc^{-1}$, $t_c = 9.05$ billion years.

The age of the universe, t_A, can be estimated from the magnitude-redshift data (Narlikar 1983, p 114) (with two signs changed to reflect contraction),

$$t_A = \frac{-1}{H_o}\left[\frac{1}{2q_o - 1} + \frac{q_o}{(2q_o - 1)^{3/2}}\cos^{-1}\frac{1 - q_o}{q_o}\right]. \qquad (37)$$

A value for $\cos^{-1}$ corresponding to the fourth quadrant must be used. For $H_o = -72.10\, km\, s^{-1} Mpc^{-1}$ and $q_o = 0.51$, $t_A = 1.54 \times 10^4$ billion years.

For $q_o = 1/2$, equation 37 gives an age of $t_A = \infty$ as it should for a flat universe. While the Pantheon data makes a persuasive case that our universe is closed, nearly flat, and very old, it does not give definitive answers to the questions "How flat is the universe?" and "How old is the universe?"

6. Velocity of light

Einstein (1907)[7] concluded that the velocity of light in special relativity, c, is reduced in an uniformly accelerated coordinate system to c^*, the local coordinate velocity of light. The ratio c/c^* is relative vacuum permittivity ε, equation 3.

For Schwarzschild geometry ε is given by equation 7. Far from the mass m, the local Schwarzschild coordinate velocity is $c^* \approx c$. At the Schwarzschild radius, where $r = 2km/c^2$, the local Schwarzschild coordinate velocity is $c^* = 0$.

For Friedmann geometry $\varepsilon(t) = a(t)$. Equation 3 gives

$$c^*(t)a(t) \;=\; c^*(t_o)a(t_o), \tag{38}$$

where $a(t_o)$ is the radius of the universe when the velocity of light is $c^*(t_o) = c = 2.998 \times 10^{10}$ cm/sec.

At the Big Bang (when $a(t) = 0$), the local coordinate velocity $c^*(t)$ was infinite before dropping to its current value c at 9.05×10^9 years, before reaching a minimum at full expansion, and then increasing back toward infinity again at collapse.

The symmetry of the Friedmann cycloid is used to equate the velocity and radius during the expansion period to the current collapsing data for H_o, q_o, and $t_o = t_c$ derived from H_o. Equations 9 then give ψ_o, $a(t_o)$ and α. This is illustrated in Figure 5.

Note that the same values on the scale are used for the velocity of light and the radius of the universe but that the units are different. This makes it easier to compare light velocity and universe size at a given time.

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[7] English translation (Einstein 1989, p 252)

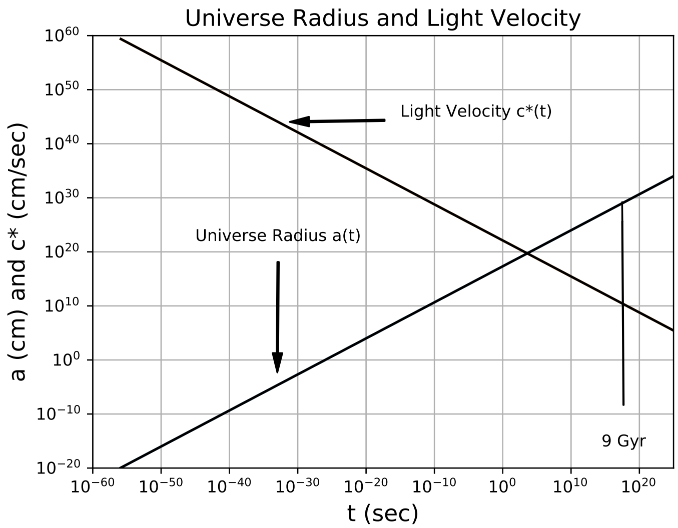

Fig. 5.— Radius and Velocity vs Time for the early Friedmann universe when it was expanding. Note the different units but the same values on the scale for the velocity of light $c^*(t)$ and the radius of the universe $a(t)$. This graph assumes $H_o = -72.10\, km\, s^{-1} Mpc^{-1}$ and $q_o = 0.51$. $t = 0$ at the Big Bang.

7. Mathematics and Physics

The mathematics of general relativity isn't a physical theory until mathematical concepts such as $g_{\mu\nu}$ and x^μ are linked by axioms to specific physical measurements. Albert Einstein took this step, just as he did for special relativity, by asserting that measurements made with rigid meter sticks and balance clocks are equivalent to the mathematical distances and times of general relativity. Assuming a rigid meter stick is equivalent to assuming that atoms never change. Even as he did this Einstein had qualms about his choices.

From Einstein's 1921 Nobel Lecture:

> . . . it would be logically more correct to begin with the whole of the laws and . . . to put the unambiguous relation to the world of experience last instead of already fulfilling it in an imperfect form for an artificially isolated part, namely the space-time metric. We are not, however, sufficiently advanced in our knowledge of Nature's elementary laws to adopt this more perfect method without going out of our depth. (Einstein 1967, p 483)

It is intriguing that it was Einstein who discovered vacuum permittivity depends on gravity. In 1907, there was no general relativity, no Bohr atom, and no clear understanding of photons.

When these theories were later in place, the connection provided by vacuum permittivity between spacetime curvature and atomic structure was overlooked. Einstein (1949, p 685) knew that the "tools for measurement do not lead an independent existence alongside of the objects implicated by the field-equations." What he did not realize was that the solution was already in his 1907 paper and that there was no need of "going out of our depth" to create the more complete general relativity he wanted, where the "tools for measurement" depend on spacetime exactly as "other objects implicated by the field-equations."

Schrödinger (1939) published his seminal discovery that every quantum wavelength expands and contracts in proportion to the radius of a closed Friedmann universe. Schrödinger argued that if spacetime is curved as general relativity requires, then its effects on quantum processes must not be dismissed without careful investigation. Using the equations of relativistic quantum mechanics, Schrödinger found that the plane-wave eigenfunctions characteristic of flat spacetimes are replaced in the curved spacetime of the closed Friedmann universe by wave functions with wavelengths that are directly proportional to the Friedmann radius.

This means that every eigenfunction changes wavelength as the radius of the universe changes. The quantum systems they describe change as well. In an expanding universe quantum systems expand. In a contracting universe they contract. The assumption is often made that small quantum systems are isolated and that their properties remain constant as the Friedmann universe evolves. This assumption is incompatible with relativistic quantum mechanics and with the curved spacetime of general relativity as Schrödinger proved (Sumner & Sumner 2000).

These changes in quantum systems may equivalently be viewed as a logical consequence of the fact that the energy and momentum of "isolated systems" are not conserved. Energy and momentum change when the spacetime curvature of the universe changes. Schrödinger (1956, p 58) wrote:

> In an expanding space *all momenta decrease* . . . for bodies acted on by no other forces than gravitation . . . This simple law has an even simpler interpretation in wave mechanics: all wavelengths, being inversely proportional to the momenta, simply expand with space. [8]

In a contracting space, the opposite is true. *All momenta increase* and all wavelengths, being inversely proportional to the momenta, simply contract with space.

Schrödinger had a deep understanding of both wave mechanics and general relativity. Like most physicists, Schrödinger "knew" Hubble redshift meant that the universe is expanding, a hangover from the pre-relativistic interpretations of redshifts originally made by Slipher (1917) and Hubble (1929) who tentatively assumed that all galactic redshifts are solely Doppler shifted photons. It

[8] Pauli (1958, p 220) made the same observation.

is interesting to speculate how long it would have taken Schrödinger to correctly interpret Hubble redshift if he had asked himself the question: "Would the changes in atoms and photons that I found change my interpretation of Hubble redshift?"

Feynman (1967, p 55) was correct when he observed that "Physics is not mathematics, and mathematics is not physics ... mathematicians prepare abstract reasoning that's ready to be used if you will only have a set of axioms about the real world ..." Assuming that meter sticks are made of atoms that never change does not belong in that set. Assuming that the speed of light is a constant doesn't belong either.

8. Conclusions

Vacuum permittivity, a measure of the strength of electric fields and light velocity in a vacuum, changes with the spacetime curvature of general relativity. This changes atomic energy levels, photon wavelengths, and the velocity of light. For Schwarzschild geometry, redshifts derived including atomic energy shifts exactly reproduce mathematical results from standard analyses assuming time dilation. While far from the mass light velocity today is nearly c, it goes to zero at the Schwarzschild radius. For Friedmann geometry a comparison of photons emitted long ago to those emitted today predicts that Hubble redshifts result from a universe accelerating in collapse. This is confirmed by the Pantheon redshift data where no modifications to general relativity or to Friedmann's 1922 assumptions are necessary to explain Hubble redshift. Assuming changes in both atomic emissions and photon wavelengths and varying only H_o and q_o gives H_o = -72.10 $\pm$ 0.75 km s^{-1} Mpc^{-1} and $1/2 < q_o < 0.51$. The average data error is 0.1418. For these fit parameters the standard deviation is 0.1515. The changes in atoms and photons derived here agree with Schrödinger's 1939 conclusion that quantum wave functions expand and contract with the radius of a closed Friedmann universe. The velocity of light is inversely proportional to vacuum permittivity which is proportional to the radius of the Friedmann universe. Light velocity was infinite at the Big Bang, dropped to its current value c at 9.05×10^9 years on its way to a minimum at full expansion. It will be infinite again 9.05×10^9 years from now.

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The Python program curve_fit from scipy.optimize was used with equations (35) and (36) to analyze the Pantheon data.

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