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ON THE VARIATION OF VACUUM PERMITTIVITY IN GENERAL RELATIVITY

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ABSTRACT

Vacuum permittivity, the measure of strength of electric fields in a vacuum, is a scalar function which depends on the curvature of spacetime. Changes in vacuum permittivity are important for many investigations. The focus here is on the required change in interpretation of gravitational redshifts. When curvature changes, photon wavelengths and atomic emissions both change. Atomic emissions shift at least twice as much as photon wavelengths do. Both must be included to correctly interpret redshift. For Schwarzschild geometry, redshift equations which include both shifts give a deeper physical understanding and exactly reproduce the well proven mathematical results of conventional analyses based on time dilation. For Friedmann geometry, a comparison of photons emitted long ago to those emitted today predicts that Hubble redshifts result from a universe accelerating in collapse. This mathematical prediction is compared to supernovae redshift data from Davis et al. and the SCPUnion2.1 compilation. For the Davis et al. data set of 156 supernovas the best fit was found to be $H_o = -66.5\,km\,s^{-1}Mpc^{-1}$ and $q_o = 1/2 + (0.001)$. The average data error is 0.231 and for these fit parameters the standard deviation is 0.234. The quality of fit for the SCPUnion2.1 compilation of 580 supernovas is similar, with $H_o = -70.2\,km\,s^{-1}Mpc^{-1}$ and $q_o = 1/2 + (0.001)$. Only H_o and q_o were varied to fit these data. No dark energy is needed when atomic shifts are included. High-z redshift observations up to 11.9 show that the universe is at least 2000 billion years old. This is more than a hundred times greater than a typical star's lifetime suggesting that most dark matter is likely the residue of stellar and galactic evolution. The time until collapse is estimated to be 9.6 billion years. The changes in atoms and photons derived here agree with Schrödinger's discovery that quantum wave functions expand and contract with the radius of a closed Friedmann universe.

Keywords: cosmology: cosmological parameters — cosmology: dark energy — cosmology: dark matter — cosmology: theory — gravitation

1. INTRODUCTION

Einstein (1907)[1] summarized the status of special relativity and its implications in the two years since its publication. At the end of his survey he concluded with a speculative section on "The Principle of Relativity and Gravitation." He considered a uniformly accelerated coordinate system and assumed that locally it is equivalent to a gravitational field. Einstein determined that Maxwell's equations in the accelerated coordinate system (and hence in a gravitational field) are exactly the same as they are in the inertial coordinate systems of special relativity, except that "The principle of the constancy of the velocity of light does not hold ... the velocity of light in the gravitational field is a function of place ..." (Einstein 1911)[2].

In Maxwell's equations, vacuum permittivity, ε, is the scalar that determines the speed of light and the strength of electrical fields. Einstein's discovery means that both the wavelengths of photons and the wavelengths of photons emitted by atoms change with gravity in special relativity. Møller (1952), Landau and Lifshitz (1975), and Sumner (1994) showed that vacuum permittivity changes with spacetime curvature in general relativity.

Two exact solutions to the theory of general relativity without a cosmological constant (Einstein 1915) are examined. One is the static solution by Schwarzschild (1916) for a spherical mass in an otherwise empty universe. The other is by Friedmann (1922) for a closed, matter-filled, homogeneous universe. Friedmann's solution rapidly expands from a singularity, then slows until it reaches a maximum size before accelerating back to a singularity.

In Schwarzschild spacetime, ε is a function of the distance r from a central mass. At long distances, $\varepsilon(r)$ is close to its flat space value and increases as r gets smaller. Atomic sizes and the wavelengths of the photons they emit change with ε. Wavelengths of photons also change with ε. The redshift equation derived here using general relativity and the traditional one obtained using time dilations are mathematically identical despite making different initial assumptions. Their weak field approximations are equivalent to an equation derived using special relativity and the equivalence principle.

In Friedmann spacetime, ε is directly proportional to the Friedmann radius so it changes with time. As the size of the Friedmann universe evolves, the changing strength of the electrical force between charges shifts atomic energy levels, changing the wavelengths of emitted light. The wavelengths of photons also change in time but by a smaller amount. This difference in evolution of atoms and photons reverses the interpretation of Hubble redshift.

The Friedmann universe is now collapsing, a result confirmed by modern supernova redshift observations. The changes in atoms and photons derived here agree with the conclusion of Schrödinger (1939) that quantum wave

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[1] English translation (Einstein 1989, p 252)
[2] English translation (Einstein 1993, p 385)

functions expand and contract proportionally to the radius of a closed Friedmann universe.

2. MATHEMATICAL MODELS

2.1. *Einstein's Solution*

In his study of Maxwell's equations in an uniformly accelerated coordinate system, Einstein (1907)[3] concluded that the velocity of light in special relativity, c, is reduced to c^*, the local coordinate velocity of light in the accelerated system. In an accelerated system which corresponds locally to the gravitational field of a point mass, Einstein (1989, p 310) found

$$c^* = c\left(1 + \frac{\Phi}{c^2}\right), \tag{1}$$

where Φ is the Newtonian gravitational potential

$$\Phi = -\frac{k\,m}{r}. \tag{2}$$

m is the mass of the object creating the gravitational field at a distance r. k is the gravitational constant.

The connection between Einstein's result, equation (1), and the strength of the electrical field comes from the definition of relative vacuum permittivity ε,

$$\varepsilon = \frac{c}{c^*}. \tag{3}$$

Combining equations (1), (2), and (3) gives Einstein's value for ε[4]

$$\varepsilon\,(r) = \frac{1}{\left(1 - \dfrac{km}{rc^2}\right)}. \tag{4}$$

2.2. *Schwarzschild Solution*

The Schwarzschild solution in general relativity may be written (Landau and Lifshitz 1975, p 301)

$$ds^2 = \left(1 - \frac{2km}{rc^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \dfrac{2km}{rc^2}\right)} - - r^2(d\theta^2 + sin^2\theta\, d\phi^2). \tag{5}$$

Møller (1952, p 308) and Landau and Lifshitz (1975, p 258) studied the effects of curved spacetime on Maxwell's equations. Both proved that in a static gravitational field the electromagnetic field equations take the form of Maxwell's phenomenological equations in a medium at rest with

$$\varepsilon(r) = \frac{1}{\sqrt{g_{00}}}. \tag{6}$$

g_{00} is the time component of a static metric tensor $g_{\mu\nu}$.[5]

Einstein's pre-general relativity result, equation (4), is the first approximation to the exact relativistic equations (5) and (6),

$$\varepsilon(r) = \frac{1}{\sqrt{g_{00}}} = \frac{1}{\sqrt{\left(1 - \dfrac{2km}{rc^2}\right)}} \approx \frac{1}{\left(1 - \dfrac{km}{rc^2}\right)}. \tag{7}$$

2.3. *Friedmann Solution*

Friedmann (1922) published a closed universe solution to Einstein's theory of general relativity without a cosmological constant. The Friedmann universe rapidly expands from a singularity, slowing until it reaches a maximum size before accelerating back to a singularity.

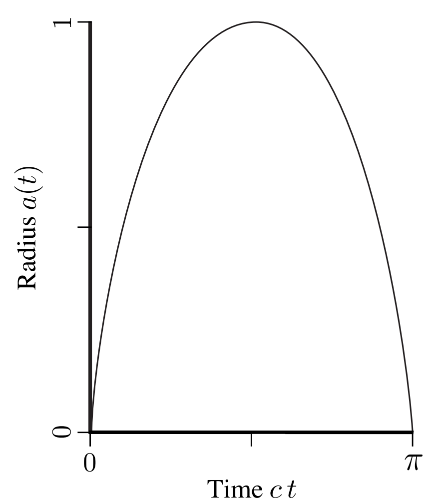

Figure 1. Friedmann's solution for a closed universe with $\alpha = 1$ in equation 8. For this cycloid scaled by α for our universe see Figure 7.

Friedmann assumed the metric,

$$ds^2 = c^2dt^2 - - a^2(t)\left[\frac{dr^2}{(1-r^2)} + r^2\left(d\theta^2 + sin^2\theta\, d\phi^2\right)\right], \tag{8}$$

and homogeneous, incoherent matter, conserved in amount, and exerting negligible pressure. His solution is the cycloid shown in Figure 1,

$$a = \frac{\alpha}{2}(1 - \cos\psi) \qquad c\,t = \frac{\alpha}{2}(\psi - \sin\psi), \tag{9}$$

where α is a constant and $0 \le \psi \le 2\pi$ (Tolman 1934).

Sumner (1994) studied Maxwell's equations in this Friedmann universe and found that ε changes in time with spacetime curvature,

$$\varepsilon\,(t) = a(t). \tag{10}$$

$a(t)$ is radius of the Friedmann universe defined above with $\alpha = 1$.

[3] English translation (Einstein 1989, p 252)

[4] Vacuum permittivity is defined by NIST to be constant with the value $\varepsilon_o = 8.854187817\ldots x10^{-12}$ Farads/meter. But as Einstein showed, vacuum permittivity is not constant. When the numerical value for vacuum permittivity for a curved spacetime is needed, the product $\varepsilon \times \varepsilon_o$ is used with ε_o expressed in appropriate units. ε is often called relative vacuum permittivity.

[5] This result is valid for every static metric, not only the Schwarzschild metric.

2.4. *Local Mathematical Coordinates*

The effects of varying spacetime curvature in nature are commonly explained with gravitational forces or are simply ignored. The extraordinary success of the special theory of relativity is a confirmation of this approach. But spacetime is never precisely flat as ubiquitous gravity clearly shows. Flat spacetime exists only in mathematical models. Every spacetime in nature is curved.

To understand the effects of spacetime curvature on atoms and photons, a coordinate system that includes spacetime curvature is necessary. The method used by Einstein, Møller, Landau, Lifshitz, and Sumner is adopted where a local pseudo-Cartesian coordinate system is used with the vacuum permittivity $\varepsilon(x^\mu)$ determined by the general relativistic geometry at that spacetime point. Specifically,

$$ds^2 = \frac{c^2}{\varepsilon^2(x^\mu)} dt^2 - \left[dr^2 + r^2\left(d\theta^2 + sin^2\theta\, d\phi^2\right)\right]. \tag{11}$$

If the variation in $\varepsilon(x^\mu)$ in the region of interest is "insignificant", equation (11) is just the metric of special relativity with a velocity of light $c/\varepsilon(x^\mu)$. If $\varepsilon = 1$ the result is special relativity with spacetime curvature ignored.

2.5. *Changes in Atoms and Photons*

Photon wavelengths, atomic sizes, and the wavelengths of photons atoms emit change with ε. In the following equations κ (the Greek letter kappa) is to be replaced by either the radial coordinate r for Schwarzschild geometry or the time coordinate t for Friedmann geometry. The logic and math are the same for each geometry.

The Bohr radius a_o of a hydrogen atom in its ground state at κ is [6]

$$a_o(\kappa) = \frac{4\pi\varepsilon_o\, \varepsilon(\kappa)\, \hbar^2}{me^2}. \tag{12}$$

$\varepsilon_o = 8.854187817\ldots x10^{-12}$ F/m (farads per meter) is the defined value of ε_o. m is the mass of the electron, e is the charge of the electron, and $\hbar$ is Planck's constant h divided by 2π. These are assumed to remain constant as spacetime curvature changes.

The change in Bohr radius a_o as κ changes is

$$\frac{a_o(\kappa_1)}{a_o(\kappa_2)} = \frac{\varepsilon(\kappa_1)}{\varepsilon(\kappa_2)}. \tag{13}$$

The characteristic wavelength λ_e emitted by a hydrogen atom during the transition between the principle quantum numbers n_2 and n_1 is

$$\lambda_e(\kappa) = \frac{8\,\varepsilon_o^2\,\varepsilon^2(\kappa)\, h^3\, c}{me^4}\left(\frac{n_1^2\, n_2^2}{n_2^2 - n_1^2}\right). \tag{14}$$

c in equation (14) comes from the defining relationship between λ and ν, $\lambda\nu = c$.

The change in $\lambda_e(\kappa)$ as κ changes is

$$\frac{\lambda_e(\kappa_1)}{\lambda_e(\kappa_2)} = \frac{\varepsilon^2(\kappa_1)}{\varepsilon^2(\kappa_2)}. \tag{15}$$

[6] See standard texts, e.g. Leighton (1959).

Consider the Compton wavelength, λ_c, of a particle with mass m_p,

$$\lambda_c(\kappa) = \frac{h}{m_p\, c^*(\kappa)} = \frac{h\,\varepsilon(\kappa)}{m_p\, c}. \tag{16}$$

The change in $\lambda_c(\kappa)$ as κ changes is

$$\frac{\lambda_c(\kappa_1)}{\lambda_c(\kappa_2)} = \frac{\varepsilon(\kappa_1)}{\varepsilon(\kappa_2)}. \tag{17}$$

The Compton wavelength of a particle is equivalent to the wavelength of a photon of the same energy as the particle. Compton and photon wavelengths have the same $\varepsilon(\kappa)$ dependency that the Bohr radius has. The wavelength change for a photon is

$$\frac{\lambda(\kappa_1)}{\lambda(\kappa_2)} = \frac{\varepsilon(\kappa_1)}{\varepsilon(\kappa_2)}. \tag{18}$$

2.6. *Gravitational Redshift*

The following notation is used. The wavelength of a photon λ emitted at κ_1 and examined at κ_1 will be written $\lambda(\kappa_1, \kappa_1)$. The wavelength of a photon λ emitted at κ_1 and examined at κ_2 will be written $\lambda(\kappa_1, \kappa_2)$.

The traditional redshift z formula assumes that atomic emissions do not evolve, $\lambda(\kappa_2, \kappa_2) = \lambda(\kappa_1, \kappa_1)$, but that photons do (Equation (18)),

$$z = \frac{\lambda(\kappa_1, \kappa_2) - \lambda(\kappa_1, \kappa_1)}{\lambda(\kappa_1, \kappa_1)} = \frac{\varepsilon(\kappa_2)}{\varepsilon(\kappa_1)} - 1. \tag{19}$$

κ_2 is the observer's location and κ_1 is the location at the time of emission.

Since atomic emissions do evolve with spacetime geometry, a new redshift variable ζ (the Greek letter zeta) is defined to match what is done experimentally,

$$\zeta = \frac{\lambda(\kappa_1, \kappa_2) - \lambda(\kappa_2, \kappa_2)}{\lambda(\kappa_2, \kappa_2)}. \tag{20}$$

κ_2 is the observer's location and κ_1 is the location at the time of emission.

$$\zeta = \frac{\varepsilon(\kappa_1)}{\varepsilon(\kappa_2)} - 1. \tag{21}$$

2.7. *Schwarzschild Redshift*

The Schwarzschild metric describes the spacetime geometry around a spherical mass.

Substituting $\varepsilon(r)$ from equation (7) into equation (21) gives

$$\zeta = \frac{\sqrt{\left(1 - \dfrac{2km}{r_2c^2}\right)}}{\sqrt{\left(1 - \dfrac{2km}{r_1c^2}\right)}} - 1. \tag{22}$$

This equation is the same as conventional derivations using time dilation with the Schwarzschild metric. See e.g. Weinberg (1972, p 80), En.wikipedia.org (2017). All experiments that confirm equation (22) confirm the reasoning presented here.

For sources and receivers separated by small distances, the weak field approximation to this exact equation is

$$\frac{\Delta\nu}{\nu} \approx \frac{km}{r^2c^2}h, \quad (23)$$

where ν is the frequency of the photon emitted, k is the gravitational constant, m is the mass of the earth, r is the distance from the center of the earth to the source, c is the speed of light, and h is the vertical separation between source and receiver.[7]

2.8. *Friedmann Redshift*

Substituting $\varepsilon(t) = a(t)$ into equation (21) gives the redshift ζ for Friedmann geometry,

$$\zeta = \frac{a(t_1)}{a(t_2)} - 1. \quad (24)$$

Hubble redshift ($\zeta > 0$) implies $a(t_1) > a(t_2)$. The universe was larger in the past, $a(t_1)$, than it is now, $a(t_2)$. This puts us somewhere on the collapsing half of the curve in Figure 1. The logic is simple. Since Hubble shifts are red ($\zeta > 0$), the Friedmann universe is collapsing. If Hubble shifts were blue ($\zeta < 0$), the Friedmann universe would be expanding.

3. ANALYZING HUBBLE REDSHIFTS

The analysis of redshift observations must include the changes in atomic emissions in addition to the changes in photons. Astronomers measure the redshift defined by ζ, equation (24). The following derivation is similar to the one made when atomic evolution is ignored and the universe is assumed to be expanding (Narlikar 1983), but is different because ζ not z describes the observed redshift and some choices in signs are made differently when the universe is contracting (Sumner and Vityaev 2000). It is assumed that observed photons were emitted after contraction began.

The mathematical coordinate distance r to a source can be shown to be a function of the observed redshift ζ of the source and the deceleration parameter q_o in the following way.

Setting $ds = 0$ in the Friedmann metric, equation (8), gives

$$c\,dt = \frac{-a(t)\,dr}{(1 - r^2)^{1/2}}. \quad (25)$$

The source is located at the spatial coordinates $(r_1, 0, 0)$ with emission at time t_1 and the observer is at $(0, 0, 0)$ with reception at time t_2.

$$c\int_{t_1}^{t_2} \frac{dt}{a(t)} = \int_0^{r_1} \frac{dr}{(1 - r^2)^{1/2}} = \sin^{-1} r_1. \quad (26)$$

Substituting $a(t)$ and dt calculated from the Friedmann solution, equation (9), gives

$$r_1 = \sin(\psi_2 - \psi_1). \quad (27)$$

The Friedmann equation for the closed universe is (Narlikar 1983, p 113)

$$\dot{a}^2 = c^2\left(\frac{\alpha}{a} - 1\right). \quad (28)$$

[7] Will (2014, p 15) has a good explanation how this approximate equation can be derived without general relativity.

The Hubble constant H and the deceleration parameter q are defined by

$$H(t) = \frac{\dot{a}(t)}{a(t)}, \quad \frac{\ddot{a}(t)}{a(t)} = -q(t)H^2(t). \quad (29)$$

" · " indicates time derivative. H is negative and q is greater than 1/2 for a closed, collapsing universe. Present day values are denoted by H_o and q_o.

α, the constant in equations (9), may be written (Narlikar 1983, p 114)

$$\alpha = \frac{2q_o}{(2q_o - 1)^{3/2}}\,\frac{c}{|H_o|}. \quad (30)$$

Solving for ψ_2 and ψ_1 in terms of ζ and q_o and substituting into equation (27) gives

$$r_1 = \frac{(2q_o - 1)^{1/2}}{q_o}\left[\zeta - \frac{(1+\zeta)(1-q_o)}{q_o}\right] + \frac{(1-q_o)}{q_o}\left\{1 - \left[\zeta - \frac{(1+\zeta)(1-q_o)}{q_o}\right]^2\right\}^{1/2}. \quad (31)$$

The flux f of photons is related to the luminosity L of the source and to its luminosity distance D_L by the equation

$$f = \frac{L}{4\pi D_L^2}. \quad (32)$$

D_L is determined in the following way. Calculate the observed flux f by noting that L, the actual luminosity of the source, is changed by a factor of $a(t_2)/a(t_1)$ because of the apparent change of the photon's energy and changed by another factor of $a(t_2)/a(t_1)$ because of the changes in time in the local metric, equation (11). The distance to the source is $r_1\,a(t_2)$. This gives an observed flux of

$$f = \frac{L\dfrac{a^2(t_2)}{a^2(t_1)}}{4\pi r_1^2 a^2(t_2)}. \quad (33)$$

Combining equations (32) and (33) using (24) gives

$$D_L = r_1\,a(t_2)\,(1 + \zeta). \quad (34)$$

$a(t_2)$ is (Narlikar 1983, p 114)

$$a(t_2) = \frac{-c}{H_o}\,\frac{1}{(2q_o - 1)^{1/2}}. \quad (35)$$

Substituting equations (31) and (35) into (34) gives

$$D_L = \frac{-c}{H_o}\frac{(1+\zeta)}{q_o}\left\{\left[\zeta - \frac{(1+\zeta)(1-q_o)}{q_o}\right] + \frac{(1-q_o)}{(2q_o-1)^{1/2}}\left(1 - \left[\zeta - \frac{(1+\zeta)(1-q_o)}{q_o}\right]^2\right)^{1/2}\right\}. \quad (36)$$

The relationship between distance modulus (the difference between the apparent magnitude m and absolute magnitude M of a celestial object) and luminosity distance, D_L, is

$$m - M = 5\log_{10}\left(\frac{D_L}{10\,\mathrm{parsecs}}\right). \qquad (37)$$

The Hubble constant H_o (negative for the contracting half of the curve) and the deceleration parameter q_o (which must be $> 1/2$) characterizing a closed Friedmann universe are then varied to find best least-squared fits to Hubble redshift observations of ζ and $m - M$ using equations (36) and (37).

4. SUPERNOVAE REDSHIFTS

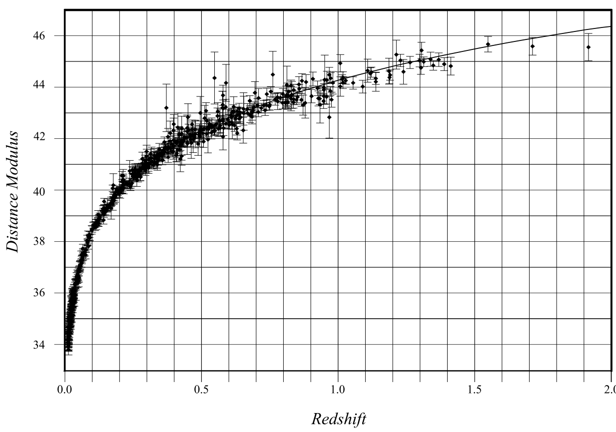

Figure 2. SCPUnion2.1 supernovae redshift data. Solid line is the least-squares fit using the Friedmann solution with the parameters $H_o = -70.2\,km\,s^{-1}Mpc^{-1}$ and $q_o = 1/2 + (0.001)$. Three SNe 1a at redshifts greater than 1.5 are also plotted (see text).

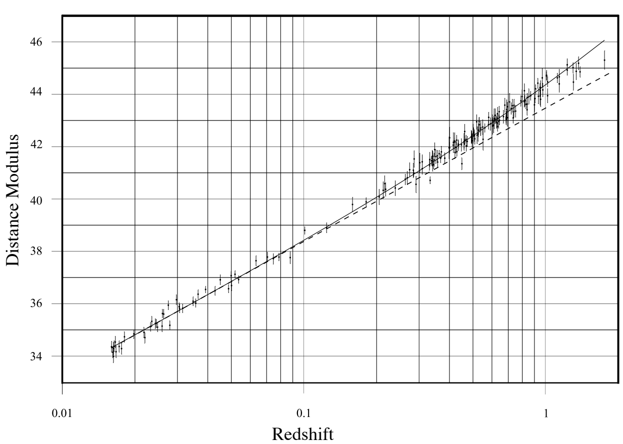

Figure 3. Supernovae redshift data from Davis et al. Solid line is the least-squares fit using the Friedmann solution with the parameters $H_o = -66.5\,km\,s^{-1}Mpc^{-1}$ and $q_o = 1/2 + (0.001)$. The straight dashed line was added to clarify the upward curve ("acceleration") of the data.

Two supernovae data sets are analyzed here, the SCPUnion2.1 compilation (Suzuki 2012)[8] and the set from Davis (2007)[9] which combines data from Wood-Vasey (2007) and Riess (2007).

For the SCPUnion2.1 data for 580 supernovas the best fit is $H_o = -70.2\,km\,s^{-1}Mpc^{-1}$ and $q_o = 1/2 + (0.001)$. This fit is shown in Figure 2. The average data error is 0.223 and for these fit parameters the standard deviation is 0.272. Although not used for fitting, three SNe 1a (with spectroscopic evidence for classification) at redshift greater than 1.5 are also plotted in Figure 2 (Rodney 2012) (Rubin 2013) (Jones 2013).

For the Davis et al. data set of 156 supernovas the best fit is $H_o = -66.5\,km\,s^{-1}Mpc^{-1}$ and $q_o = 1/2 + (0.001)$. This fit is shown in Figure 3. The average data error is 0.231 and for these fit parameters the standard deviation is 0.234.

Figure 4 shows the percentage differences from best fit for other choices of H_o and q_o. For both data sets the least-squared fits are better the closer q_o is to 1/2.

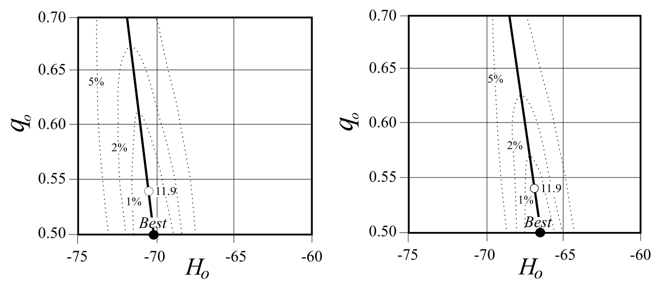

Figure 4. The fit parameter space for H_o and q_o. The solid line shows the best fit for a given choice of H_o or q_o. Percentage contours are the differences from the best fit shown by the black dot. See text for meaning of 11.9.

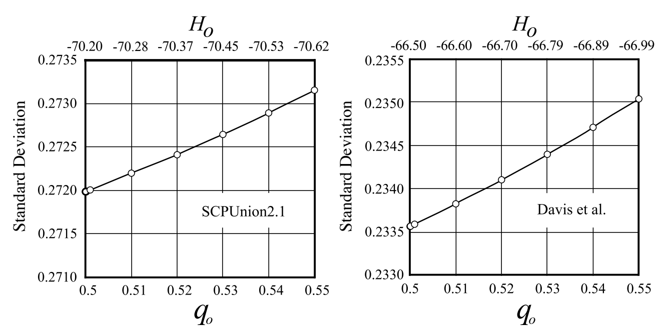

Figure 5. Standard deviation for best fits for smaller values of q_o for each data set. The corresponding best fit H_o for each q_o is indicated on the top axis. These data establish no lower limit for δ, where $q_o = 1/2 + \delta$, $\delta > 0$.

Figure 5 illustrates standard deviation for best fits for smaller values of q_o for each data set. These data establish no lower limit on δ, where $q_o = 1/2 + \delta$, $\delta > 0$. $\delta = 0.001$ is an arbitrary small number in the range where there is little further improvement in the quality of fit with smaller δ's.

[8] These data were downloaded from `http://supernova.lbl.gov/Union/figures/SCPUnion2.1_mu_vs_z.txt`

[9] These data were downloaded from `http://braeburn.pha.jhu.edu/~ariess/R06/Davis07_R07_WV07.dat`

5. AGE OF THE UNIVERSE

If q_o were known, the age of the universe, t_o, could be estimated from the magnitude-redshift data (Narlikar 1983, p 114) (with two signs changed to reflect contraction),

$$t_o = \frac{-1}{H_o}\left[\frac{1}{2q_o-1} + \frac{q_o}{(2q_o-1)^{3/2}}\cos^{-1}\frac{1-q_o}{q_o}\right]. \quad (38)$$

A value for $\cos^{-1}$ corresponding to the fourth quadrant is assumed.

Since $(2q_o - 1) \approx 0$, equation (38) cannot be applied, but it does give a way to estimate a minimum age of the universe if a maximum value $q_{o\,max}$ for q_o can be estimated. This can be done using a maximum observed redshift, ζ_{max} and assuming that the light was emitted just when the universe was at its maximum size, $a(t_m)$. $a(t_m)$ is related to today's parameters by (Weinberg 1972, p 483),

$$a(t_m) = \frac{2q_0}{2q_0-1}\, a(t_0). \quad (39)$$

Combining with equation (24) gives

$$q_{o\,max} = \frac{1+\zeta_{max}}{2\,\zeta_{max}}. \quad (40)$$

Substituting $q_{o\,max}$ and H_o in equation (38) then gives an estimate for the *minimum age* of a collapsing Friedmann universe. This is a conservative estimate since it is very likely that the light was emitted after the universe reached its maximum size.

Ellis (2013) has measured redshifts in the range of 8.6 to 11.9 utilizing a sequence of near-infrared Wide Field Camera 3 images of the Hubble Ultra Deep Field. The open circles in Figure 4 mark the maximum value 0.54 that q_0 can have and still observe a redshift of 11.9.

Figure 6 illustrates the relationship of minimum age to maximum observed redshift determined in this way for both data sets.

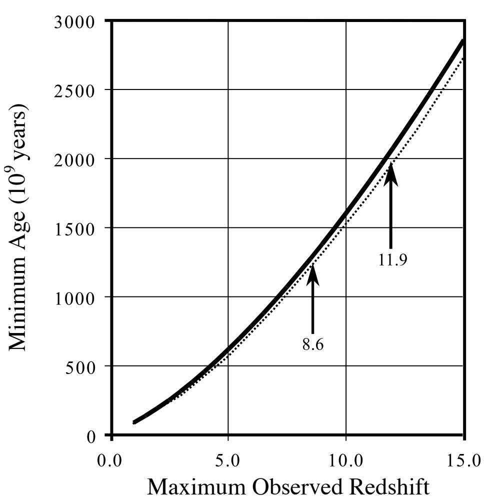

Figure 6. Minimum ages of a Friedmann universe for maximum observed redshifts. The solid line uses H_0 from Davis et al. data and the dotted line uses H_0 from SCPUnion2.1 data. The redshifts indicated are from Ellis et al.

From these redshift measurements the universe is at least 2000 billion years old.

6. TIME UNTIL COLLAPSE

$\frac{2}{3}\,|H_o|^{-1}$ estimates the time until collapse, t_c, of the Friedmann universe when q_o is close to 1/2. [10]
$H_o = -66.5\,kms^{-1}Mpc^{-1}$ gives $t_c = 9.80$ billion years. $H_o = -70.1\,kms^{-1}Mpc^{-1}$ gives $t_c = 9.30$ billion years. $t_c = 9.6$ billion years averages the uncertainties from these two data sets.

7. OUR UNIVERSE

The estimated minimum age of our universe and the time until collapse scale the Friedmann curve using equations (9) and (24). The maximum radius was 637 billion light-years, 1000 billion years ago. The current radius is estimated to be 49 billion light-years.

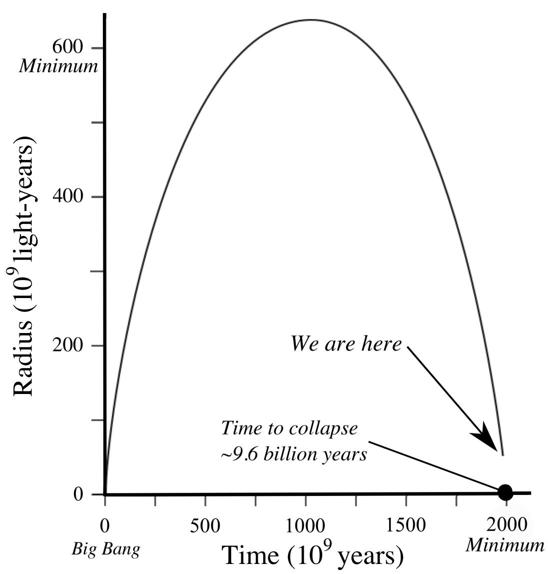

Figure 7. This represents the evolution of our universe since the big bang at time zero. It reached its maximum size 1000 billion years ago. Today there remains 9.6 billion years until collapse. Its current radius is 49 billion light-years. These values are minimum estimates based on redshift analyses in this paper and assuming that $q_o = 0.54$. See text for details.

Figure 7 illustrates this evolution. The maximum radius and current age will both be greater than shown on this graph if q_o is smaller than 0.54.

8. DARK ENERGY AND DARK MATTER

The close agreement of Friedmann theory with redshift observations makes it is unnecessary to postulate dark energy.

Dark matter must exist since visible matter is only a fraction of the mass required for a closed Friedmann universe. Since the universe is more than 100 times older than the lifetimes of typical stars, most dark matter is likely the end result of stellar and galactic evolution.

[10] This can be shown by evaluating Weinberg (1972, eq 15.3.11) in the limit as $q_o \to 1/2$ and using the symmetry of the cycloid.

9. MATHEMATICS AND PHYSICS

The mathematics of general relativity isn't a physical theory until mathematical concepts such as $g_{\mu\nu}$ and x^μ are linked by axioms to specific physical measurements. Albert Einstein took this step, just as he did for special relativity, by asserting that measurements made with rigid meter sticks and balance clocks are equivalent to the mathematical distances and times of general relativity. Assuming a rigid meter stick is equivalent to assuming that atoms never change. Even as he did this Einstein had qualms about his choice.

In his 1921 Nobel Lecture Einstein said:

> ... it would be logically more correct to begin with the whole of the laws and ... to put the unambiguous relation to the world of experience last instead of already fulfilling it in an imperfect form for an artificially isolated part, namely the space-time metric. We are not, however, sufficiently advanced in our knowledge of Nature's elementary laws to adopt this more perfect method without going out of our depth. (Einstein 1967, p 483)

It is intriguing that it was Einstein who discovered vacuum permittivity depends on gravity. In 1907, there was no general relativity, no Bohr atom, and no real understanding of photons. When these theories were later in place, the connection provided by vacuum permittivity between spacetime curvature and atomic structure was overlooked. Einstein (1949, p 685) knew that the "tools for measurement do not lead an independent existence alongside of the objects implicated by the field-equations." What he did not know was that the solution was already in his 1907 paper and that there was no need of "going out of our depth" to create the more complete general relativity he wanted, where the "tools for measurement" depend on spacetime exactly as "other objects implicated by the field-equations."

Schrödinger (1939) published his seminal discovery that every quantum wavelength expands and contracts in proportion to the Friedmann radius. Schrödinger argued that if spacetime is curved as general relativity requires, then its effects on quantum processes must not be dismissed without careful investigation. Using the equations of relativistic quantum mechanics, Schrödinger found that the plane-wave eigenfunctions characteristic of flat spacetimes are replaced in the curved spacetime of the closed Friedmann universe by wave functions with wavelengths that are directly proportional to the Friedmann radius.

This means that every eigenfunction changes wavelength as the radius of the universe changes. The quantum systems they describe change as well. In an expanding universe quantum systems expand. In a contracting universe they contract. The assumption is often made that small quantum systems are isolated and that their properties remain constant as the Friedmann universe evolves. This assumption is incompatible with relativistic quantum mechanics and with the curved spacetime of general relativity as Schrödinger showed (Sumner and Sumner 2000).

These changes in quantum systems may equivalently be viewed as a logical consequence of the fact that the energy and momentum of "isolated systems" are not conserved. Energy and momentum change when the spacetime curvature of the universe changes. Schrödinger (1956, p 58) wrote:

> In an expanding space *all momenta decrease* ... for bodies acted on by no other forces than gravitation ... This simple law has an even simpler interpretation in wave mechanics: all wavelengths, being inversely proportional to the momenta, simply expand with space.[11]

In a contracting space, the opposite is true. *All momenta increase* and all wavelengths, being inversely proportional to the momenta, simply contract with space.

Schrödinger had a deep understanding of both wave mechanics and general relativity. Like most physicists, Schrödinger "knew" Hubble redshift meant that the universe is expanding, a hangover from the pre-relativistic interpretations of redshifts originally made by Slipher (1917) and Hubble (1929) who tentatively assumed that all galactic redshifts are solely Doppler effects. It is interesting to speculate how long it would have taken Schrödinger to correctly interpret Hubble redshift if he had asked himself the question: "Would the changes in atoms and photons that I found change my interpretation of Hubble redshift?"

Feynman (1967, p 55) was correct when he observed that "Physics is not mathematics, and mathematics is not physics ... mathematicians prepare abstract reasoning that's ready to be used if you will only have a set of axioms about the real world ..." Assuming that meter sticks are rigid and atoms never change should not be axioms in that set.

10. CONCLUSIONS

Einstein assumed that gravitational fields are equivalent to uniformly accelerating coordinate systems and showed that vacuum permittivity depends on the strength of the gravitational field. Vacuum permittivity also changes with spacetime curvature, shifting the energy of atoms and photons and the interpretation of gravitational redshifts. Redshift results when blueshifted photons are compared to atomic emissions which have blueshifted more. For Schwarzschild geometry, redshifts derived this way agree with conventional derivations based on time dilations. While the approaches are different, their mathematical predictions are identical. But for Friedmann geometry, traditional theory ignoring atomic shifts does not agree with the reasoning presented here, nor does it agree with modern Hubble redshift observations. Comparisons of photons emitted long ago to those emitted today show that the Friedmann universe is collapsing, not expanding. Supernovae redshifts are explained using physics from the 1920's without invoking ad hoc dark energy. Our collapsing universe is finite and nearly flat. It is at least 2000 billion years old and will end in an estimated 9.6 billion years. Most dark matter is probably the residue of stellar and galactic evolution. The changes in atoms and photons derived here agree with Schrödinger's discovery that quantum wave functions expand and contract with the radius of a closed Friedmann universe.

[11] Pauli (1958, p 220) made the same observation.

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