

# HUBBLE REDSHIFT REVISITED

W. Q. SUMNER

ABSTRACT. In 1907 Einstein used special relativity to prove that vacuum permittivity is a function of gravity by assuming that acceleration and gravity are equivalent. Vacuum permittivity is the scalar in Maxwell's equations that determines the speed of light and the strength of electrical fields. Predictably, vacuum permittivity also changes with spacetime curvature in general relativity. When spacetime curvature changes, the wavelengths of both photons and atomic emissions shift. In Friedmann geometry, curvature changes in time. A photon today has a different wavelength than it did yesterday. Yesterday, an atom emitted a photon with a different wavelength than it emits today. Considered together, the evolution of atoms and photons reverse the interpretation of Hubble redshift. Hubble redshift implies that the Friedmann universe is closed and collapsing. During collapse, both atomic emissions and photons blueshift. Atomic emissions blueshift about twice as much as photons blueshift. This means that blueshifted photons seen in a telescope today are redder than blueshifted reference photons emitted by atoms today. With this insight, supernovae redshift observations are fit simply using the physics of Maxwell, Einstein, Bohr, and Friedmann from the 1920's. There is no need to postulate dark energy. Supernovae redshift data imply that the universe is very nearly flat and will collapse in about 9.6 billion years. High- $z$  redshift observations up to 11.9 suggest that the universe is at least 2000 billion years old. This is more than a hundred times greater than a typical star's lifetime. It is probable that most dark matter is the residue of stellar evolution. The changes in atoms and photons derived here confirm Schrödinger's 1939 proof that quantum wave functions expand and contract proportionally to the radius of a closed Friedmann universe.

## 1. INTRODUCTION

In 1907 Einstein [1] [2, p 252] summarized the status of special relativity and its implications in the two years since its publication. At the end of this survey he concluded with a speculative section on "The Principle of Relativity and Gravitation." He considered an uniformly accelerated coordinate system and assumed that locally it is equivalent to a gravitational field. Einstein concluded that Maxwell's equations in the accelerated coordinate system (and hence in a gravitational field) are exactly the same as they are in the inertial coordinate systems of special relativity, except that "The principle of the constancy of the velocity of light does not hold . . . the velocity of light in the gravitational field is a function of place . . ." [3] [4, p 385]

In Maxwell's equations, vacuum permittivity is the scalar that determines the speed of light and the strength of electrical fields. Einstein's discovery means that both the wavelengths of photons and the wavelengths of photons emitted by atoms change with gravity in special relativity and with spacetime curvature in general relativity.

In a Friedmann universe, vacuum permittivity is directly proportional to the Friedmann radius [5] and is therefore a function of time. As the size of the universe evolves, the changing strength of the electrical force between charges shifts atomic energy levels, changing the wavelengths of emitted light. This shift in photon emission due to the evolution of electrical attraction in the atom is about twice as large as the evolutionary shift in photon wavelength. The evolution of atoms and photons together reverse the interpretation of Hubble redshift to imply that the Friedmann universe is closed and is now collapsing.

Schrödinger [6] provided another perspective on the evolution of atoms and photons. He showed that the plane-wave eigenfunctions characteristic of flat spacetime are replaced in the curved spacetime of the closed Friedmann universe by eigenfunctions which have wavelengths that are directly proportional to the Friedmann radius. This means that eigenfunctions change wavelength as the radius of the universe changes and the quantum systems they describe do as well. In an expanding universe quantum systems expand. In a contracting universe they contract. Photons and atoms both change with the radius of the Friedmann universe.

Since atoms as well as photons change in time, it is necessary to rederive the connection between measured Hubble redshifts, the Hubble constant  $H_o$ , the deceleration parameter  $q_o$ , and the distances to the sources. When this is done, modern accelerating supernovae redshifts are beautifully fit by varying only  $H_o$  and  $q_o$ . These redshift data imply that the universe is very nearly flat ( $q_o \approx \frac{1}{2}$ ) and will collapse in about 9.6 billion years. High- $z$  redshift observations up to 11.9 suggest that the universe is at least 2000 billion years old. This is more than a hundred times greater than a typical star's lifetime. This makes it likely that most dark matter is the residue of stellar evolution.

## 2. THE VARIATION OF VACUUM PERMITTIVITY IN THE GRAVITATIONAL FIELD

In his study of Maxwell's equations in an uniformly accelerated coordinate system, Einstein concluded that the velocity of light in special relativity,  $c$ , is reduced to  $c^*$ , the local coordinate velocity of light in the accelerated system. In an accelerated system corresponding locally to the gravitational field of a point mass, Einstein [2, p 310] found

$$(1) \quad c^* = c [1 + (\Phi/c^2)],$$

where  $\Phi$  is the Newtonian gravitational potential,

$$(2) \quad \Phi = -\frac{kM}{r}.$$

$M$  is the mass of the object creating the gravitational field at a distance  $r$ .  $k$  is the gravitational constant.

The connection between Einstein's result, equation (1), and the strength of the electrical field follows from the relationship of relative vacuum permittivity  $\varepsilon$  to  $c^*$  and  $c$ ,

$$(3) \quad \varepsilon = \frac{c}{c^*}.$$

Combining equations (1), (2), and (3) gives Einstein's value for  $\varepsilon$ ,

$$(4) \quad \varepsilon(r) = \frac{1}{(1 - kM/c^2r)}.$$

Møller [7, p 308] and Landau & Lifshitz [8, p 258] studied the effects of curved spacetime on Maxwell's equations. Both proved that in a static gravitational field the electromagnetic field equations take the form of Maxwell's phenomenological equations in a medium at rest with

$$(5) \quad \varepsilon(r) = \frac{1}{\sqrt{g_{00}}}.$$

$g_{\mu\nu}$  is the metric tensor. Einstein's pre-general relativity result, equation (4), is the first approximation to the exact equation (5) for the Schwarzschild metric [9] where  $g_{00} = (1 - 2kM/c^2r)$ .

Sumner [5] studied Maxwell's equations in the closed Friedmann universe without a cosmological constant and also found that  $\varepsilon$  varies with spacetime curvature. This may be verified directly by calculating the coordinate speed of light  $c^*$  for the Friedmann metric, equation (8), and substituting in equation (3). This reproduces Sumner's result,

$$(6) \quad \varepsilon(t) = a(t).$$

$a(t)$  is the radius of the closed Friedmann universe defined by equations (8) and (9).

The electrical field  $|\mathbf{E}|$  of a charge  $q$  at a distance  $r_q$  is<sup>1</sup>

$$(7) \quad |\mathbf{E}| = \frac{1}{4\pi\varepsilon_o\varepsilon(t)} \frac{|q|}{r_q^2}.$$

$|\mathbf{E}|$  is a function not only of the distance  $r_q$  from the charge  $q$  but also changes in time in Friedmann geometry. For Schwarzschild geometry,  $\varepsilon(r)$  replaces  $\varepsilon(t)$  and  $\mathbf{E}$  changes with  $r$ , the distance from the gravitating source.

### 3. FRIEDMANN GEOMETRY

Friedmann [10] found a closed universe solution to Einstein's [11] theory of general relativity without a cosmological constant. The Friedmann universe rapidly expands from a singularity, slowing until it reaches a maximum size before accelerating back to a singularity.

Friedmann's solution is found by solving Einstein's equations assuming the metric,

$$(8) \quad ds^2 = c^2 dt^2 - a^2(t) \left[ \frac{dr^2}{(1 - r^2)} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right].$$

The fluid in the model is assumed to be incoherent matter, conserved in amount and exerting negligible pressure. The solution is the cycloid shown in Figure 1,

$$(9) \quad a = \frac{\alpha}{2}(1 - \cos\psi) \quad ct = \frac{\alpha}{2}(\psi - \sin\psi),$$

where  $\alpha$  is a constant and  $0 \leq \psi \leq 2\pi$  [12].

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<sup>1</sup>SI equations use the constant flat-space vacuum permittivity  $\varepsilon_o$  multiplied by the relative vacuum permittivity  $\varepsilon$ .  $\varepsilon$  is a scalar field and does not change with coordinate transformations [5].

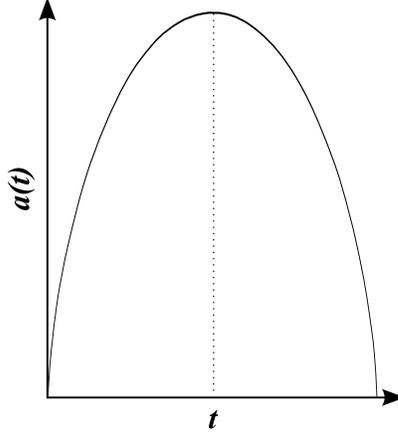


FIGURE 1. Friedmann's solution for a closed universe.

## 4. ATOMS AND PHOTONS

The extraordinary successes of special relativity are evidence that the curvature caused by the earth and by the universe is very small. But spacetime is not precisely flat as ubiquitous gravity, Hubble redshift, and careful research have shown. Flat spacetime exists only in mathematical models. Every spacetime in nature is curved.

To understand the effects of spacetime curvature on atoms and photons, a coordinate system that includes spacetime curvature is necessary. Since the deviations from special relativity are small, the method used by Einstein, Møller, and Landau & Lifshitz is adopted. Specifically, a local pseudo-Cartesian coordinate system with the local velocity of light  $c^*$  given by general relativity is used,<sup>2</sup>

$$(10) \quad ds^2 = \frac{c^2}{a^2(t)} dt^2 - [dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)].$$

Physics equations that are valid in flat spacetime are then used with  $c^* = c/a(t)$  and  $\varepsilon(t) = a(t)$  to interpret evolutionary changes in atoms and photons.

The Bohr radius  $a_o$  of a hydrogen atom in its ground state at time  $t$  is<sup>3</sup>

$$(11) \quad a_o(t) = \frac{4\pi\varepsilon_o \varepsilon(t) \hbar^2}{me^2}.$$

The change in Bohr radius  $a_o$  as time  $t$  changes is

$$(12) \quad \frac{a_o(t_1)}{a_o(t_0)} = \frac{\varepsilon(t_1)}{\varepsilon(t_0)}.$$

<sup>2</sup>In equation (8) it is an extremely good approximation in this context to assume  $g_{11} = -a^2(t)$ , or equivalently,  $(1 - r^2) \approx 1$ . The values of  $r$  of interest are the order of the size of a hydrogen atom and its photons. For scale,  $r = 1$  corresponds to the "size" of the universe.

<sup>3</sup>See any standard text, e.g. Leighton [13]. In the following equations  $\varepsilon(t)$  may be replaced by  $a(t)$ .

The characteristic wavelength  $\lambda_e$  emitted by a hydrogen atom during the transition between the principle quantum numbers  $n_2$  and  $n_1$  is

$$(13) \quad \lambda_e(t) = \frac{8 \varepsilon_o^2 \varepsilon^2(t) h^3 c}{m e^4} \left( \frac{n_1^2 n_2^2}{n_2^2 - n_1^2} \right).$$

The change in  $\lambda_e(t)$  as  $t$  changes is

$$(14) \quad \frac{\lambda_e(t_1)}{\lambda_e(t_0)} = \frac{\varepsilon^2(t_1)}{\varepsilon^2(t_0)}.$$

$c$  in equation (13) comes from the defining relationship between  $\lambda$  and  $\nu$ ,  $\lambda \nu = c$ .

Consider the Compton wavelength,  $\lambda_c$ , of a particle with mass  $m_p$ ,

$$(15) \quad \lambda_c(t) = \frac{h}{m_p c^*(t)} = \frac{h \varepsilon(t)}{m_p c}.$$

The change in  $\lambda_c(t)$  as  $t$  changes is

$$(16) \quad \frac{\lambda_c(t_1)}{\lambda_c(t_0)} = \frac{\varepsilon(t_1)}{\varepsilon(t_0)}.$$

The Compton wavelength of a particle is equivalent to the wavelength of a photon of the same energy as the particle. Compton and photon wavelengths have the same  $t$  dependency that the Bohr radius has.

The following notation is used. The wavelength of a photon  $\lambda$  emitted at  $t_1$  and examined at the time of emission  $t_1$  will be written  $\lambda(t_1, t_1)$ . The wavelength of a photon  $\lambda$  emitted at  $t_1$  and examined at a later time  $t_0$  will be written  $\lambda(t_1, t_0)$ . With this notation, the wavelength change for a photon is

$$(17) \quad \frac{\lambda(t_1, t_1)}{\lambda(t_1, t_0)} = \frac{\varepsilon(t_1)}{\varepsilon(t_0)}.$$

## 5. HUBBLE REDSHIFT

The Hubble redshift shift equation for Friedmann geometry is derived using equation (17) to describe the changes in photons and equation (14) to describe the changes in atomic emissions. A photon emitted at a time  $t_1$  and observed at  $t_0$  is compared to a reference photon emitted and observed at time  $t_0$ . Equation (6),  $\varepsilon(t) = a(t)$ , gives the relative vacuum permittivity for Friedmann geometry.

From equation (17), the wavelength of the emitted photon at  $t_1$ ,  $\lambda(t_1, t_1)$ , is related to its wavelength at  $t_0$ ,  $\lambda(t_1, t_0)$ , by

$$(18) \quad \lambda(t_1, t_0) = \frac{\varepsilon(t_0)}{\varepsilon(t_1)} \lambda(t_1, t_1).$$

From equation (14), a photon emitted at  $t_1$ ,  $\lambda_e(t_1) = \lambda(t_1, t_1)$ , is related to one emitted at  $t_0$ ,  $\lambda_e(t_0) = \lambda(t_0, t_0)$ , by

$$(19) \quad \lambda(t_1, t_1) = \frac{\varepsilon^2(t_1)}{\varepsilon^2(t_0)} \lambda(t_0, t_0).$$

Combining equations (6), (18), and (19) gives the change in wavelength of a photon emitted at  $t_1$  and observed at  $t_0$ ,

$$(20) \quad \frac{\lambda(t_1, t_0)}{\lambda(t_0, t_0)} = \frac{\varepsilon(t_1)}{\varepsilon(t_0)} = \frac{a(t_1)}{a(t_0)}.$$

The Hubble redshift observed,  $\lambda(t_1, t_0) > \lambda(t_0, t_0)$ , implies that  $a(t_1) > a(t_0)$ . The closed Friedmann universe is contracting.

This reversal in interpretation of Hubble redshift from expansion to contraction is a direct result of the different rates of change of energy for photons and atoms. In an expanding universe, atomic emissions out redshift photons giving a relative blueshift. In a contracting universe, atomic emissions out blueshift photons giving a relative redshift.<sup>4</sup>

## 6. ANALYZING HUBBLE REDSHIFTS

The analysis of redshift observations must be modified to include changes in atomic emissions as well as changes in photons, beginning with the mathematical definition of redshift.

Redshift  $z$  is traditionally defined

$$(21) \quad z = \frac{\lambda_{obs}(t_0) - \lambda}{\lambda},$$

where  $\lambda$  is the assumed constant atomic emission and  $\lambda_{obs}(t_0)$  is the observed wavelength of this emission from a distant source. With the assumptions that photons evolve,  $\lambda_{obs}(t_0) = (a_0/a_1)\lambda$ , but that atomic emissions do not,

$$(22) \quad z = \frac{a(t_0)}{a(t_1)} - 1.$$

$a(t_0)$  is the current radius and  $a(t_1)$  was the radius at the time of emission.

Define a new redshift variable  $k$  as

$$(23) \quad k = \frac{\lambda(t_1, t_0) - \lambda(t_0, t_0)}{\lambda(t_0, t_0)},$$

where  $\lambda(t_1, t_0)$  is the wavelength observed today from a distant source and  $\lambda(t_0, t_0)$  is the wavelength emitted today by a reference atom. Astronomers measure the redshift defined by  $k$ .  $k$  is equivalent to

$$(24) \quad k = \frac{a(t_1)}{a(t_0)} - 1,$$

using equation (20).

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<sup>4</sup>The same mathematical logic applies for Schwarzschild geometry.  $r$  may be substituted for  $t$  and  $\varepsilon(r)$  substituted for  $\varepsilon(t)$  in equation (20) to give the spectral shift that would be observed at  $r_0$  of a photon emitted at  $r_1$ .

$$\frac{\lambda(r_1, r_0)}{\lambda(r_0, r_0)} = \frac{\varepsilon(r_1)}{\varepsilon(r_0)} \quad \text{with} \quad \varepsilon(r) = \frac{1}{\sqrt{(1 - 2kM/c^2r)}}.$$

For  $r_0 > r_1$  the observed shift is red. For  $r_1 > r_0$ , the observed shift is blue.

The mathematical coordinate distance  $r$  to a source is a function of the observed redshift  $k$  of the source and the deceleration parameter  $q_o$ . The following derivations are similar to the ones made when atomic evolution is ignored and the universe is assumed to be expanding [14], but differ because  $k$  not  $z$  describes the observed redshift and some choices in signs are made differently when the universe is contracting [15]. It is assumed that the observed photons were emitted after contraction began.

Setting  $ds = 0$  in the Friedmann metric, equation (8), gives

$$(25) \quad c dt = \frac{-a(t) dr}{(1 - r^2)^{1/2}}.$$

The source is located at the spatial coordinates  $(r_1, 0, 0)$  with emission at time  $t_1$  and the observer is at  $(0, 0, 0)$  with reception at time  $t_0$ .

$$(26) \quad c \int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_0^{r_1} \frac{dr}{(1 - r^2)^{1/2}} = \sin^{-1} r_1.$$

Substituting  $a(t)$  and  $dt$  calculated from the Friedmann solution, equation (9), gives

$$(27) \quad r_1 = \sin(\psi_0 - \psi_1).$$

The Friedmann equation for the closed universe is [14, p 113]

$$(28) \quad \dot{a}^2 = c^2 \left( \frac{\alpha}{a} - 1 \right).$$

The Hubble constant  $H$  and the deceleration parameter  $q$  are defined by

$$(29) \quad H(t) = \frac{\dot{a}}{a}, \quad \frac{\ddot{a}}{a} = -q(t)[H(t)]^2.$$

$H$  is negative and  $q$  is greater than 1/2 for a closed, collapsing universe. Present day values are denoted by  $H_o$  and  $q_o$ .

$\alpha$ , the constant in equation (9), may be written

$$(30) \quad \alpha = \frac{2q_o}{(2q_o - 1)^{3/2}} \frac{c}{|H_o|}.$$

Solving for  $\psi_0$  and  $\psi_1$  in terms of  $k$  and  $q_o$  and substituting into equation (27) gives

$$(31) \quad r_1 = \frac{(2q_o - 1)^{1/2}}{q_o} \left[ k - \frac{(1 + k)(1 - q_o)}{q_o} \right] + \frac{(1 - q_o)}{q_o} \left\{ 1 - \left[ k - \frac{(1 + k)(1 - q_o)}{q_o} \right]^2 \right\}^{1/2}.$$

Luminosity distance  $D_L$  is related to the observed flux  $f$  and the intrinsic luminosity  $L$  of the source by the equation

$$(32) \quad f = \frac{L}{4\pi D_L^2}.$$

The flux observed can be calculated by noting that the luminosity is decreased by a factor of  $a(t_0)/a(t_1)$  because of the apparent decrease of the photon's energy and decreased by another factor of  $a(t_0)/a(t_1)$  because of the changes in time in the local metric, equation (10). The distance to the source is  $r_1 a(t_0)$ . This gives an observed flux of

$$(33) \quad f = \frac{L \frac{a^2(t_0)}{a^2(t_1)}}{4\pi r_1^2 a^2(t_0)}.$$

Comparing equations (32) and (33) gives

$$(34) \quad D_L = r_1 a(t_0) (1 + k),$$

where equation (24) has been used. Combining equations (31) and (34) and noting that

$$(35) \quad a(t_0) = \frac{-c}{H_o} \frac{1}{(2q_o - 1)^{1/2}},$$

gives the desired result

$$(36) \quad D_L = \frac{-c}{H_o} \frac{(1+k)}{q_o} \left\{ \left[ k - \frac{(1+k)(1-q_o)}{q_o} \right] + \frac{(1-q_o)}{(2q_o - 1)^{1/2}} \left( 1 - \left[ k - \frac{(1+k)(1-q_o)}{q_o} \right]^2 \right)^{1/2} \right\}.$$

The relationship between distance modulus (the difference between the apparent magnitude  $m$  and absolute magnitude  $M$  of a celestial object) and luminosity distance,  $D_L$ , is

$$(37) \quad m - M = 5 \log_{10} \left( \frac{D_L}{10 \text{ parsecs}} \right).$$

The Hubble constant  $H_o$  (negative for the contracting half of the curve) and the deceleration parameter  $q_o$  (which must be  $> \frac{1}{2}$ ) characterizing a closed Friedmann universe are then varied to find best least-squared fits to Hubble redshift observations of  $m - M$  and  $k$  using equations (36) and (37).

## 7. SUPERNOVAE REDSHIFTS

Two supernovae data sets are analyzed here, the SCPUnion2.1 compilation<sup>5</sup> [16] and the set from Davis et al.<sup>6</sup> [17] which combines data from Wood-Vasey et al. [18] and Riess et al. [19]. For the SCPUnion2.1 data the best fit was found to be  $H_o = -70.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and  $q_o = \frac{1}{2} + (0.001)$ . This fit is shown in Figure 2. The average data error is 0.223 and for these fit parameters the standard deviation is 0.272. Although not used for fitting, three SNe 1a (with spectroscopic evidence for classification) at redshift greater than 1.5 are also plotted in Figure 2 [20] [21] [22].

<sup>5</sup>These data were downloaded from [http://supernova.lbl.gov/Union/figures/SCPUnion2.1\\_mu\\_vs\\_z.txt](http://supernova.lbl.gov/Union/figures/SCPUnion2.1_mu_vs_z.txt)

<sup>6</sup>These data were downloaded from [http://braeburn.pha.jhu.edu/~ariess/R06/Davis07\\_R07\\_WV07.dat](http://braeburn.pha.jhu.edu/~ariess/R06/Davis07_R07_WV07.dat)

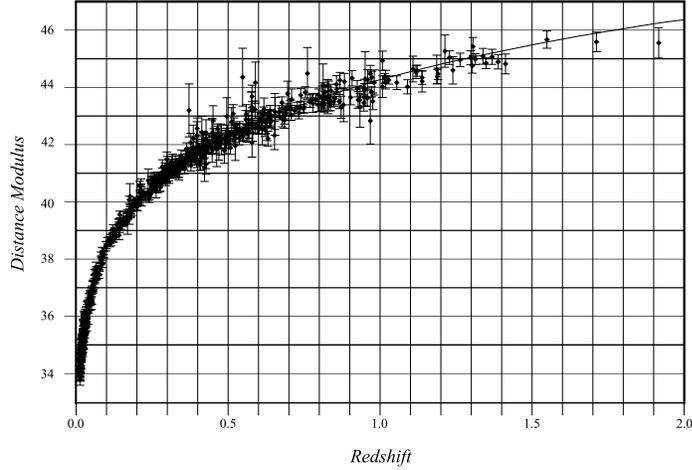


FIGURE 2. SCPUnion2.1 supernovae redshift data. Solid line fit using the Friedman solution with the parameters  $H_o = -70.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and  $q_o = \frac{1}{2} + (0.001)$ . Three SNe 1a at redshifts greater than 1.5 are also plotted (see text).

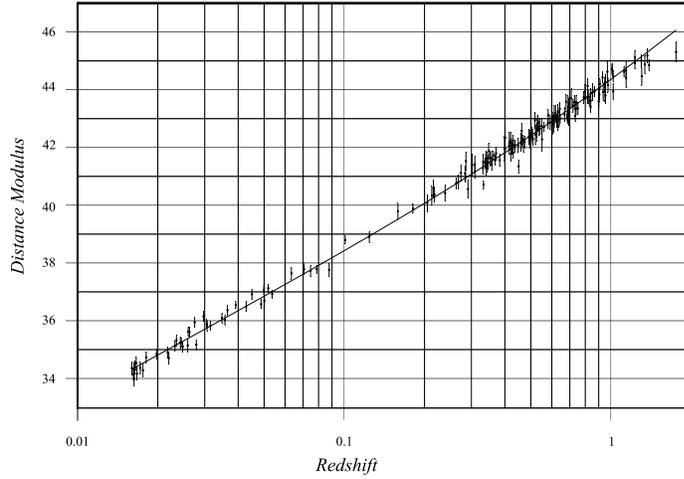


FIGURE 3. Supernovae redshift data from Davis et al. Solid line fit using the Friedman solution with the parameters  $H_o = -66.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and  $q_o = \frac{1}{2} + (0.001)$ .

For the Davis et al. data set the best fit was found to be  $H_o = -66.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and  $q_o = \frac{1}{2} + (0.001)$ . This fit is shown in Figure 3. The average data error is 0.231 and for these fit parameters the standard deviation is 0.234.

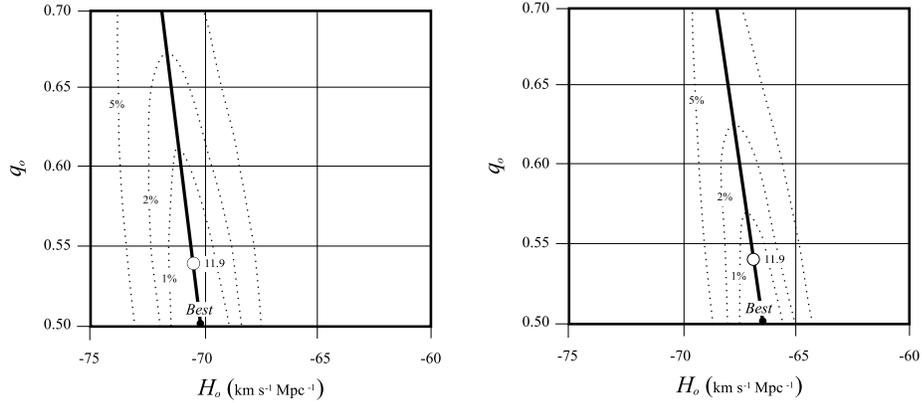


FIGURE 4. The fit parameter space for  $H_o$  and  $q_o$ . The black line shows the best fit for a given choice of  $H_o$  or  $q_o$ . Percentage contours are the differences from the best fit shown by the black dot. See text for meaning of 11.9.

Figure 4 shows the percentage differences from best fit for other choices of  $H_o$  and  $q_o$ . For both data sets the least-squared fits are better the closer  $q_o = \frac{1}{2} + \delta$  is to  $\frac{1}{2}$ .  $\delta = 0.001$  is near the value where improvement in fit diminishes as  $\delta$  is further reduced. No lower limit for  $\delta$  was found. Figure 5 illustrates average deviation for best fits for smaller values of  $q_o$  for each data set. Both redshift data sets imply that the universe is very nearly flat.

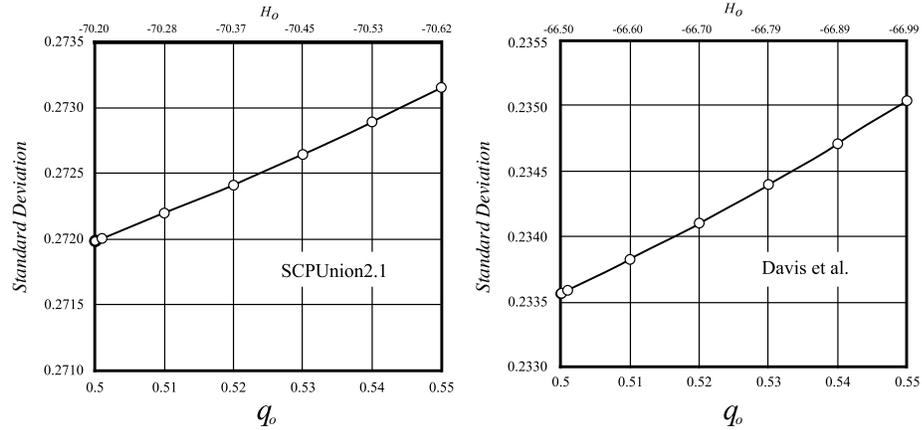


FIGURE 5. Average deviation for best fits for smaller values of  $q_o$  for each data set. The corresponding best fit  $H_o$  for each  $q_o$  is indicated on the top axis.

## 8. MINIMUM AGE OF THE UNIVERSE

The age of the universe may be estimated from the magnitude-redshift data in the usual manner [14, p 114], except one sign must be changed to reflect contraction,

$$(38) \quad t_o = \frac{-1}{H_o} \left[ \frac{1}{2q_o - 1} + \frac{q_o}{(2q_o - 1)^{3/2}} \cos^{-1} \frac{1 - q_o}{q_o} \right].$$

A value for  $\cos^{-1}$  corresponding to the fourth quadrant is assumed.

Since  $(2q_o - 1) \approx 0$ , equation (38) cannot be used directly. But equation (38) will give an estimate for the minimum age of the universe if a maximum value for  $q_o$  is known. A maximum  $q_o$  may be estimated using a maximum observed redshift,  $k_{max}$ , and assuming that the light was emitted just when the universe was at its maximum size. Combining equation (24) for  $k$  with an equation giving the maximum of the curve  $a(t_m)$  in terms of today's parameters [23, p 483],

$$(39) \quad a(t_m) = \frac{2q_o}{2q_o - 1} a(t_o),$$

gives  $q_{o\ max}$ ,

$$(40) \quad q_{o\ max} = \frac{1 + k_{max}}{2k_{max}}.$$

Substituting  $q_{o\ max}$  and  $H_o$  in equation (38) then gives a robust estimate for the *minimum age* of a collapsing Friedmann universe. This is only a *minimum age estimate* since there is no way of knowing whether the light was emitted when the universe was at its maximum size.

Figure 6 illustrates the relationship of minimum age to maximum observed redshift determined in this way for both data sets.

Ellis et al. [24] have measured redshifts in the range of 8.6 to 11.9 utilizing a new sequence of near-infrared Wide Field Camera 3 images of the Hubble Ultra Deep Field. Table 1 presents their redshift results and minimum ages calculated using  $H_o = -66.5\ km\ s^{-1}\ Mpc^{-1}$  from fitting Davis et al. [17]. The open circles in Figure 4 mark the maximum value  $q_o$  can have and still observe a redshift of 11.9. This value is  $q_o = 0.54$ .

From these redshift measurements it is likely that the universe is greater than 2000 billion years old.

## 9. DARK ENERGY, DARK MATTER, TIME UNTIL COLLAPSE

The close agreement of Friedmann theory and redshift measurements makes it unnecessary to postulate dark energy.

Dark matter must exist since visible matter is only a small fraction of the mass required to close the Friedmann universe. Most dark matter is likely the end result of stellar evolution since the universe is more than 100 times older than the lifetimes of typical stars. But nothing in this analysis precludes other kinds of dark matter.

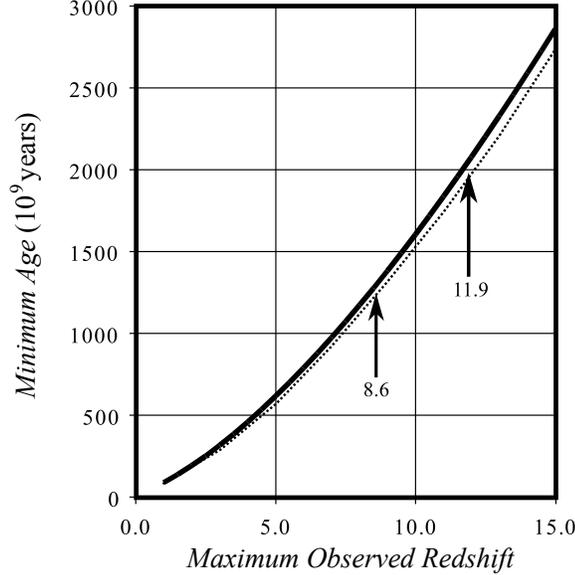


FIGURE 6. Minimum ages of a Friedmann universe for maximum observed redshifts. The solid line uses  $H_0$  from Davis et al. and the dotted line uses  $H_0$  from SCPUnion2.1.

TABLE 1.  $z > 8.5$  Candidates and Implied Minimum Ages for Friedmann Universes

ID	RA	Decl.	$z_{\text{SED}}(\pm 1\sigma)$	Age( $\pm 1\sigma$ ) 10 <sup>9</sup> years
UDF12-3954-6284	3:32:39.54	-27:46:28.4	11.9 $^{+0.3}_{-0.5}$	2046 $^{+74}_{-122}$
UDF12-4106-7304	3:32:41.06	-27:47:30.4	9.5 $^{+0.4}_{-0.8}$	1485 $^{+89}_{-173}$
UDF12-4265-7049	3:32:42.65	-27:47:04.9	9.5 $^{+0.4}_{-0.7}$	1485 $^{+89}_{-152}$
UDF12-3921-6322	3:32:39.21	-27:46:32.2	8.8 $^{+0.4}_{-0.2}$	1333 $^{+86}_{-42}$
UDF12-4344-6547	3:32:43.44	-27:46:54.7	8.8 $^{+0.5}_{-0.5}$	1333 $^{+108}_{-105}$
UDF12-3895-7114	3:32:38.95	-27:47:11.4	8.6 $^{+0.8}_{-0.6}$	1291 $^{+172}_{-125}$
UDF12-3947-8076	3:32:39.47	-27:48:07.6	8.6 $^{+0.2}_{-0.2}$	1291 $^{+42}_{-42}$

$\frac{2}{3} |H_o|^{-1}$  estimates the time until collapse of the Friedmann universe when  $q_o$  is near 0.5.  $H_o = -66.5 \text{ kms}^{-1} \text{ Mpc}^{-1}$  gives 9.80 billion years.  $H_o = -70.1 \text{ kms}^{-1} \text{ Mpc}^{-1}$  gives 9.30 billion years.

## 10. DISCUSSION

“Physics is not mathematics, and mathematics is not physics . . . mathematicians prepare abstract reasoning that’s ready to be used if you will only have a set of axioms about the real world . . .” Feynman [25, p 55].

The mathematics of general relativity isn’t a physical theory until mathematical concepts such as  $g_{\mu\nu}$  and  $x^\mu$  are linked by axioms to specific laboratory methods that measure things like distance and time. Albert Einstein took this step just as he did for special relativity by asserting that measurements made with rigid meter sticks and balance clocks are equivalent to the mathematical distances and times of general relativity. Assuming a rigid meter stick is equivalent to assuming that atoms never change. Even as he did this Einstein had qualms about this choice.

In his 1921 Nobel Lecture Einstein said:

. . . it would be logically more correct to begin with the whole of the laws and . . . to put the unambiguous relation to the world of experience last instead of already fulfilling it in an imperfect form for an artificially isolated part, namely the space-time metric. We are not, however, sufficiently advanced in our knowledge of Nature’s elementary laws to adopt this more perfect method without going out of our depth. [26, p 483]

Einstein continued to have these misgivings his whole career. He wrote in 1949:

Is there such a thing as a natural object which incorporates the “natural-measuring-stick” independently of its position in four-dimensional space? The affirmation of this question made the invention of the general theory of relativity psychologically possible; however this supposition is logically not necessary. For the construction of the present theory of relativity the following is essential:

(1) Physical things are described by continuous functions, field-variables of four coordinates. As long as the topological connection is preserved, these latter can be freely chosen.

(2) The field-variables are tensor-components; among the tensors is a symmetrical tensor  $g_{ik}$  for the description of the gravitational field.

(3) There are physical objects, which (in the macroscopic field) measure the invariant  $ds$ .

If (1) and (2) are accepted, (3) is plausible, but not necessary. The construction of mathematical theory rests exclusively upon (1) and (2).

A complete theory of physics as a totality, in accordance with (1) and (2) does not yet exist. If it did exist, there would be no room for the supposition (3). For the objects used as tools for measurement do not lead an independent existence alongside of the objects implicated by the field-equations. [27, p 685]

It is intriguing that it was Einstein who discovered vacuum permittivity depends on gravity. In 1907, there was no general relativity, no Bohr atom, and no real understanding

of photons. When these theories were later in place, the connection provided by vacuum permittivity between spacetime curvature and atomic structure was overlooked. Einstein knew that the “tools for measurement do not lead an independent existence alongside of the objects implicated by the field-equations.” What he did not know was that the solution was already in his 1907 paper and that there was no need of “going out of our depth” to create the more complete general relativity he wanted, where the “tools for measurement” depend on spacetime exactly as “other objects implicated by the field-equations.”

In 1990 Sumner rediscovered that vacuum permittivity changes with spacetime curvature and began exploring some of its consequences for Friedmann geometry. At that time Sumner was unaware of Einstein’s 1907 paper and was unaware of Schrödinger’s 1939 seminal discovery that every quantum wavelength expands and contracts in proportion to the Friedmann radius.

Schrödinger [6] argued that if spacetime is curved as general relativity demands, then its effects on quantum processes must not be dismissed without careful investigation. Using the equations of relativistic quantum mechanics, Schrödinger found that the plane-wave eigenfunctions characteristic of flat spacetimes are replaced in the curved spacetime of the closed Friedmann universe by wave functions that are not precisely flat. They have wavelengths that are directly proportional to the Friedmann radius.

This means that every eigenfunction changes wavelength as the radius of the universe changes. The quantum systems they describe do as well. In an expanding universe quantum systems expand. In a contracting universe they contract. The assumption is often made that small quantum systems are isolated and that their properties remain constant as the Friedmann universe evolves. This assumption is incompatible with relativistic quantum mechanics and with the curved spacetime of general relativity as Schrödinger proved [28].

These changes in quantum systems may equivalently be viewed as a logical consequence of the fact that the energy and momentum of “isolated systems” are not conserved. Energy and momentum change when the spacetime curvature of the universe changes. Schrödinger wrote:

“In an expanding space *all momenta decrease* . . . for bodies acted on by no other forces than gravitation . . . This simple law has an even simpler interpretation in wave mechanics: all wavelengths, being inversely proportional to the momenta, simply expand with space.”<sup>7</sup> [30, p 58]

In a contracting space, the opposite is true. *All momenta increase* and all wavelengths, being inversely proportional to the momenta, simply contract with space.

Schrödinger had a deep understanding of both wave mechanics and general relativity. Like most physicists, Schrödinger “knew” Hubble redshift meant that the universe is expanding, a hangover from the pre-relativistic interpretations of redshifts originally made by Slipher [31] and Hubble [32] who tentatively assumed that all galactic redshifts are solely Doppler effects. It is interesting to speculate how long it would have taken Schrödinger to correctly interpret Hubble redshift if he had asked himself the question, “Would the

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<sup>7</sup>Pauli [29, p 220] made the same observation.

changes in atoms and photons I found change the interpretation of Hubble redshift?" A good guess is that it would have taken Schrödinger only a few heartbeats.

Feynman was correct when he noted that "Physics is not mathematics, and mathematics is not physics." You do need the right "set of axioms about the real world." Assuming meter sticks are rigid and atoms never change is not one of them.

## 11. CONCLUSIONS

Einstein's assumption that gravitational fields are equivalent to uniformly accelerating coordinate systems led to his discovery that the coordinate speed of light and vacuum permittivity are functions of the strength of the gravitational field and ultimately led him to general relativity. The dependence of vacuum permittivity on local spacetime curvature implies that the strength of electrical fields also depends on spacetime location, influencing the sizes and the energies of both atoms and photons. This must be factored into every interpretation of physical measurements made for every spacetime geometry. For Friedmann geometry, a comparison of the spacetime curvature when photons were emitted long ago to the curvature today proves that Hubble redshifts result from a Friedmann universe that is contracting. The changes in atoms and photons derived here confirm Schrödinger's proof that quantum wave functions expand and contract proportionally to the radius of a closed Friedmann universe.

The physics of Maxwell, Einstein, Bohr, and Friedmann from the 1920's explain modern Hubble redshift measurements. There is no need to postulate dark energy. Our universe is finite, collapsing, and very nearly flat. It began in a big bang at least 2000 billion years ago and will end in about 9.6 billion years. Most dark matter likely results from stellar evolution since the age of the universe is much greater than stellar lifetimes.

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