

# Distance and Time Measurements in Friedmann Universes

William Q. Sumner<sup>1</sup>

## ABSTRACT

Analyzing distance and time measurements in Friedmann geometry leads to four conclusions: (1) The physical length of a rod measured by a meter stick does not change in time; (2) The physical length of a rod measured by the travel time of light using an atomic clock is increasing about one part in  $15 \times 10^9$  per year; (3) Neither method measures mathematical distance; and (4) Atomic clocks do not measure mathematical time.

## 1. Introduction

Consider a right triangle with sides of 1 meter and a hypotenuse  $\sqrt{2}$  meters. This mathematical triangle does not exist in nature. No angle is exactly 90 degrees and no physical measurement ever gives the irrational number  $\sqrt{2}$ . At best, distances measured in nature are reasonable numerical approximations to those defined in mathematics. 1.414213562373095 is neither numerically nor logically the same as  $\sqrt{2}$ .

Time is also defined differently in mathematics than it is in the laboratory. Stressing this distinction Newton (1729) wrote

Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external . . . measure of duration by the means of motion, which is commonly used instead of true time . . . It may be, that there is no such thing as an equable motion, whereby time may be accurately measured.

Newton's "absolute time" is simply mathematical time and his "relative time" is simply time measured by some natural process. Newton makes the same clear distinction for space (Newton 1729).

---

<sup>1</sup>Isba for Physical Problems, Kittitas, WA USA

I do not define time, space, place, and motion, as being well known to all. Only I must observe, that the common people conceive those quantities under no other notions but from the relation they bear to sensible objects.

Distance and time measurements in nature result from following clear procedures using specified instruments. Mathematical definitions for distance and time are freely chosen. If the math model used reflects nature and correspondences made between math and observation reasonably match, math can be used to predict natural phenomena. Observations of nature in turn can be used to refine the mathematical model.

Euclidean geometry is the quintessential example of this use of mathematics. Mathematical physics supplies many more. Here the mathematical solution to Albert Einstein’s general theory of relativity found by Alexandr Friedmann is used for examining the tools and processes of mensuration.

## 2. Friedmann Universe

The geometry of the universe is assumed to be described by Friedmann’s (1922) closed solution to Einstein’s (1916) equations of general relativity

$$ds^2 = c^2 dt^2 - a^2(t) \left[ \frac{dr^2}{(1-r^2)} + r^2 (d\vartheta^2 + \sin^2\vartheta d\varphi^2) \right], \quad (1)$$

here approximated by

$$ds^2 = c^2 dt^2 - a^2(t) [dr^2 + r^2 (d\vartheta^2 + \sin^2\vartheta d\varphi^2)]. \quad (2)$$

The closed Friedmann solution describes a dynamic universe born in fire and explosively expanding to a maximum size and then collapsing.  $a(t)$ , a cycloid, measures the size of the universe. This big bang solution came from tying geometry to the energy of billions of galaxies each with billions of stars. Friedmann’s solution is supported by many observations, including measurements of the Hubble redshift of light from distant stars and galaxies.

In a seminal paper Erwin Schrödinger (1939) proved that quantum wave functions coevolve with the curved spacetime of the closed Friedmann universe. “In an expanding space *all momenta decrease* ... for bodies acted on by no other forces than gravitation ... This simple law has an even simpler interpretation in wave mechanics: all wavelengths, being inversely proportional to the momenta, simply expand with space.” (Schrödinger 1956).

Schrödinger’s results were confirmed by an examination of electrical fields in Friedmann geometry (Sumner 1994). There it was shown that both photons and atoms coevolve and

only a contracting, closed Friedmann universe exhibits Hubble redshift. Schrödinger chose the closed solution for theoretical reasons, a choice that is experimentally confirmed by Hubble redshift.

The mathematical distance,  $l_m(t)$ , between two points  $(t, r_1, 0, 0)$  and  $(t, r_2, 0, 0)$  is, from equation (2),

$$l_m(t) = a(t) |r_2 - r_1|. \quad (3)$$

$l_m(t)$  is proportional to  $a(t)$ , changing as the Friedmann universe evolves.

$t$  is mathematical time and “... of itself, and from its own nature, flows equably without relation to anything external ...” (Newton 1729).

### 3. Distance and Time Measurements

Schrödinger and Sumner assumed like Einstein that (1) spacetime is curved and (2) every mathematical description of nature should be consistent with curved spacetime geometry. Schrödinger’s conclusion “all wavelengths ... simply expand with space” confirms the changes in photon and particle momenta known observationally and theoretically from other reasoning in general relativity.

Two physical definitions of distance and one for time are considered.

One definition for the meter is the length between two marks on a reference platinum-iridium bar. Meter sticks calibrated to this platinum-iridium bar can be used to measure distance.

The meter can also be defined as the distance travelled by light in free space in  $1/299,792,458$  of a second.

The unit of time, the second, is defined as the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.

Are these physical definitions for distance and time to be identified with the mathematical distances and times in Friedmann spacetime? Einstein (1949) pondered this question from the inception of general relativity until his death. He wrote:

Can a spectral line be considered as a measure of a “proper time” (*EigenZeit*)  $ds$  ( $ds^2 = g_{ik}dx_i dx_k$ ), (if one takes into consideration regions of cosmic dimensions)? Is there such a thing as a natural object which incorporates the “natural-

measuring-stick” independently of its position in four-dimensional space? The affirmation of this question made the invention of the general theory of relativity psychologically possible; however this supposition is logically not necessary. For the construction of the present theory of relativity the following is essential:

(1) Physical things are described by continuous functions, field-variables of four co-ordinates. As long as the topological connection is preserved, these latter can be freely chosen.

(2) The field-variables are tensor-components; among the tensors is a symmetrical tensor  $g_{ik}$  for the description of the gravitational field.

(3) There are physical objects, which (in the macroscopic field) measure the invariant  $ds$ .

If (1) and (2) are accepted, (3) is plausible, but not necessary. The construction of mathematical theory rests exclusively upon (1) and (2). A *complete* theory of physics as a totality, in accordance with (1) and (2) does not yet exist. If it did exist, there would be no room for the supposition (3). For the objects used as tools for measurement do not lead an independent existence alongside of the objects implicated by the field-equations.

Schrödinger’s proof that all quantum wave functions coevolve with spacetime geometry showed that “tools for measurement do not lead an independent existence alongside of the objects implicated by the field-equations” and that there is “no room for the supposition (3)”. The sizes of atoms and their characteristic emissions depend on  $a(t)$ . Einstein’s reasoning, once again, was faultless.

Consider the two physical definitions for distance and the effects of the change in atoms over time. Assume for simplicity that the time taken to make a measurement is much shorter than the time between consecutive measurements, i.e. measurements are done at a given time.

Measure the length of a rod,  $l_p(t_1)$ , with a meter stick at time  $t_1$ . When the measurement is repeated at  $t_2$ , the length of the meter stick has changed by  $a(t_2)/a(t_1)$  according to Schrödinger. The rod, since it is also made of atoms, changes exactly the same amount as the meter stick. No change in dimension consequently is detected.

If it is assumed that the ends of the rod remain at the same mathematical coordinates,  $(t, r_1, 0, 0)$  and  $(t, r_2, 0, 0)$ , the mathematical distance according to equation (3) changes exactly like the meter stick. While the mathematical distance and the length of the rod change by the same amount, neither change is detected with a meter stick. Mathematical distance is not measured by a meter stick.

Now measure the length of the rod by timing a pulse of light,  $l_l(t_1)$ , at the time  $t_1$ . Here time is assumed to be measured by counting *Ticks* or periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom. It is logically simpler to use a single atomic clock and time the round trip from  $r_1$  to  $r_2$  and back.

From equation (2), the mathematical time for the round trip,  $\tau(t_1)$  is

$$\tau(t_1) = \frac{2l_m(t_1)}{c} = \frac{2a(t_1)|r_2 - r_1|}{c}. \quad (4)$$

The mathematical time at a later time,  $\tau(t_2)$  is

$$\tau(t_2) = \frac{2l_m(t_2)}{c} = \frac{2a(t_2)|r_2 - r_1|}{c}. \quad (5)$$

The ratio of times reflects the change in Friedmann geometry

$$\frac{\tau(t_2)}{\tau(t_1)} = \frac{a(t_2)}{a(t_1)}. \quad (6)$$

The mathematical time multiplied by the frequency of the hyperfine transition,  $\nu(t_1)$ , gives the clock reading

$$Ticks(t_1) = \tau(t_1)\nu(t_1). \quad (7)$$

$l_l(t_1)$  is proportional to  $Ticks(t_1)$ . The clock reading at a later time is

$$Ticks(t_2) = \tau(t_2)\nu(t_2). \quad (8)$$

$l_l(t_2)$  is proportional to  $Ticks(t_2)$ .

The frequency of the atom's hyperfine transition coevolves with Friedmann geometry (Sumner 1994)

$$\nu(t_2) = \frac{a^2(t_1)}{a^2(t_2)} \nu(t_1). \quad (9)$$

Combining gives

$$\frac{Ticks(t_2)}{Ticks(t_1)} = \frac{l_l(t_2)}{l_l(t_1)} = \frac{a(t_1)}{a(t_2)}. \quad (10)$$

A comparison of the ratio of mathematical times, equation (6), to the ratio for atomic clock times, equation (10), shows that mathematical time is not measured by an atomic clock in an evolving Friedmann universe. The ratios change in opposite directions.

Physical distances defined by measuring the elapsed times for light travel,  $l_l$ , increase when the Friedmann universe and mathematical distances decrease. Physical distances measured with a meter stick do not change. The two definitions for physical distance do not agree and neither measure mathematical distance.

To evaluate the size of the distance discrepancies, begin with an estimate that the universe will collapse in about  $15 \times 10^9$  years (Sumner 2005). If one assumes that the size of the universe decreases linearly with time (a reasonable first approximation), equation (8) shows that  $l_l(t)$  will increase about one part in  $15 \times 10^9$  per year.

#### 4. Summary

A study of the changes in physical measurements caused by changes in Friedmann geometry shows four things: (1) The physical length of a rod measured by a meter stick does not change in time; (2) The physical length of a rod measured by the travel time of light using an atomic clock is increasing about one part in  $15 \times 10^9$  per year; (3) Neither method measures mathematical distance; and (4) Atomic clocks do not measure mathematical time.

## REFERENCES

- Einstein, A. E., 1916, Ann. d. Phys. 49, 769
- Einstein, A. E. in Paul Arthur Schilpp 1949, Albert Einstein: Philosopher-Scientist (La Salle: Open Court), p. 685
- Friedmann, A. A., 1922, Z. Phys., 10, 377
- Newton, I., 1729, Principia (1934, Berkeley: University of California Press) p. 6
- Schrödinger, E., 1939, Physica, 6, 899
- Schrödinger, E., 1956, Expanding universes (Cambridge: Cambridge University Press) p. 58
- Sumner, W. Q., 1994, ApJ, 429, 491
- Sumner, W. Q., 2005, arXiv:astro-ph/0503161v1

---

Correspondence should be addressed to [wqsumner@gmail.com](mailto:wqsumner@gmail.com)

Version: March 8, 2010